Pumping of electromagnetic energy by multiple total reflections from gainy media

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We propose an experiment for strong light amplification at multiple total reflections from active gaseous media.

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I. INTRODUCTION

Light reflection from an interface between two media is determined by the wave equation and the boundary conditions, which follow from Maxwell's equations. The wave equations for electric, $\boldsymbol{E}$ and magnetic, $\boldsymbol{H}$, fields in a homogeneous medium with constant μ and ϵ are

$$\Delta\boldsymbol{E}(\boldsymbol{r},t)=-\frac{\mu\epsilon}{c^2}\frac{\partial^2}{\partial t^2}\boldsymbol{E}(\boldsymbol{r},t),\qquad \Delta\boldsymbol{H}(\boldsymbol{r},t)=-\frac{\mu\epsilon}{c^2}\frac{\partial^2}{\partial t^2}\boldsymbol{H}(\boldsymbol{r},t). \tag{1}$$

Both equations have plain wave solutions

$$\boldsymbol{E}(\boldsymbol{r},t)=\boldsymbol{E}\exp(i\boldsymbol{k}\cdot\boldsymbol{r}-i\omega t),\qquad \boldsymbol{H}(\boldsymbol{r},t)=\boldsymbol{H}\exp(i\boldsymbol{k}\cdot\boldsymbol{r}-i\omega t), \tag{2}$$

where $k^2=\epsilon\mu k_0^2$, $k_0=\omega/c$ and c is the speed of light in vacuum. The fields $\boldsymbol{E}$ and $\boldsymbol{H}$ are not independent. They are related to each other by equations

$$\boldsymbol{H}=\frac{1}{\mu\omega}[\boldsymbol{k}\times\boldsymbol{E}],\qquad \boldsymbol{E}=-\frac{1}{\epsilon\omega}[\boldsymbol{k}\times\boldsymbol{H}], \tag{3}$$

and, if $|\boldsymbol{E}|=1$, the length of $\boldsymbol{H}$ is $|\boldsymbol{H}|=\sqrt{\varepsilon/\mu}=1/Z$, where $Z=\sqrt{\mu/\varepsilon}$ is called the medium impedance.

If space consists of two halves with different $\epsilon_{1,2}$ and $\mu_{1,2}$, the wave equations in them (1) are different, and their solutions should be matched at the interface. The matching conditions follow from the Maxwell equations. They require continuity of the components $\boldsymbol{E}_{\|}(\boldsymbol{r},t)$, $\boldsymbol{H}_{\|}(\boldsymbol{r},t)$ parallel to the interface, and $\epsilon(\boldsymbol{n}\cdot\boldsymbol{E}(\boldsymbol{r},t))$, $\mu(\boldsymbol{n}\cdot\boldsymbol{H}(\boldsymbol{r},t))$, perpendicular to it, where $\boldsymbol{n}$ is a unit normal vector. The wave function in presence of the interface is

$$\boldsymbol{\psi}(\boldsymbol{r},t)=\Theta(z<0)\Big(e^{i\boldsymbol{k}_1\cdot\boldsymbol{r}-i\omega t}\boldsymbol{\psi}_1+e^{i\boldsymbol{k}_r\cdot\boldsymbol{r}-i\omega t}\boldsymbol{\psi}_r\rho\Big)+\Theta(z>0)e^{i\boldsymbol{k}_2\cdot\boldsymbol{r}-i\omega t}\boldsymbol{\psi}_2\tau, \tag{4}$$

where the term $\exp(i\boldsymbol{k}_1\cdot\boldsymbol{r}-i\omega t)\boldsymbol{\psi}_1$ with the wave vector $\boldsymbol{k}_1$ describes the plain wave incident on the interface from medium 1, factors $\boldsymbol{\psi}_i=\boldsymbol{E}_i+\boldsymbol{H}_i$ $(i=1,r,2)$ denote sum of electric and magnetic polarization vectors, $\boldsymbol{k}_{r,2}$ are wave vectors of the reflected and transmitted waves, ρ, τ are the reflection and transmission amplitudes respectively, and $\Theta(z)$ is the step function, which is equal to unity when inequality in its argument is satisfied, and to zero otherwise.

The wave vectors $\boldsymbol{k}_{r,2}$ are completely determined by $\boldsymbol{k}_1$. They are determined uniquely by the constants ϵ_i, μ_i, and by the fact that $k_0=\omega/c$ and the part $\boldsymbol{k}_{\|}$ of the wave vectors parallel the interface must be identical for all the waves. In the following we assume that the medium 1 is lossless, i.e. $\epsilon_1\mu_1$ is real, therefore all the components of $\boldsymbol{k}_1$ are also real.

The normal component $k_{2\perp}$ of the refracted wave is

$$k_{2\perp}=\sqrt{\epsilon_2\mu_2k_0^2-\boldsymbol{k}_{\|}^2}=\sqrt{k_{1\perp}^2-(\epsilon_1\mu_1-\epsilon_2\mu_2)k_0^2}, \tag{5}$$

or it can be represented as

$$k_{2\perp}=\sqrt{\epsilon k_1^2-\boldsymbol{k}_{\|}^2}=\sqrt{n^2k_1^2-\boldsymbol{k}_{\|}^2}, \tag{6}$$

where $n=\sqrt{\epsilon}$ is the refractive index, and we introduced relative permittivity $\epsilon=\epsilon_2\mu_2/\epsilon_1\mu_1$.

The amplitudes ρ and τ are well known from textbooks (see [1], for example). They are calculated differently for TE-wave, when the incident electric field is polarized perpendicularly to the plane of incidence, i.e. parallel to the interface (it is usually denoted as $\boldsymbol{E}_s$), and for TH-field, when the incident electric field is polarized incide the plane of incidence (it is usually denoted as $\boldsymbol{E}_p$). For both of this cases we have well known Fresnel formulas

$$\rho_s=\frac{\mu_2k_{1\perp}-\mu_1k_{2\perp}}{\mu_2k_{1\perp}+\mu_1k_{2\perp}},\qquad \rho_p=\frac{\epsilon_2k_{1\perp}-\epsilon_1k_{2\perp}}{\epsilon_2k_{1\perp}+\epsilon_1k_{2\perp}}, \tag{7}$$

and $\tau_{s,p}=1+\rho_{s,p}$. In the following we for simplicity assume that $\mu_2=\mu_1=1$, so $\mu_{1,2}$ are excluded from all our formulas.

From (6) it follows that for lossless media when $0 < \epsilon < 1$ is real, the incident wave, for which $\boldsymbol{k}_\parallel$ is within $nk_1 \leq |\boldsymbol{k}_\parallel| \leq k_1$, is totally reflected from the interface. This happens because

$$k_{2\perp} = iK''_{2\perp} \equiv i\sqrt{k_\parallel^2 - \epsilon k_1^2}, \tag{8}$$

thus the factor $\exp(ik_{2\perp}z) = \exp(-K''_{2\perp}z)$ of the wave $\exp(i\boldsymbol{k_2 r})$ exponentially decays, i.e. the refracted wave becomes an evanescent one. Therefore, the energy does not flow inside the medium 2, and due to the energy conservation it must be totally reflected into medium 1.

If the medium 2 is lossy or gainy, the constant ϵ is a complex quantity $\epsilon = \epsilon' \pm i\epsilon''$, with positive ϵ' and ϵ''. In this case outside the total internal reflection (TIR) region ($|\boldsymbol{k}_\parallel|^2 \ll \epsilon' k_1^2$) we have $k_{2\perp} = k'_{2\perp} \pm ik''_{2\perp}$, where for small ϵ'' ($\epsilon'' k_1^2 \ll \epsilon' k_1^2 - |\boldsymbol{k}_\parallel|^2$)

$$k'_{2\perp} \approx \sqrt{\epsilon' k_1^2 - |\boldsymbol{k}_\parallel|^2}, \quad k''_{2\perp} \approx \epsilon'' \frac{k_1^2}{2k'_{2\perp}}. \tag{9}$$

In the TIR regime, $k'_{2\perp}$ in Eq. 9 transforms into $iK''_{2\perp}$, where $K''_{2\perp} \approx \sqrt{|\boldsymbol{k}_\parallel|^2 - \epsilon' k_1^2}$, and $k''_{2\perp}$ transforms to

$$k''_{2\perp} \to -iK'_{2\perp} = \epsilon'' \frac{k_1^2}{2iK''_{2\perp}}. \tag{10}$$

Therefore, at TIR $k_{2\perp} = \pm K'_{2\perp} + iK''_{2\perp}$, where

$$K'_{2\perp} = \epsilon'' \frac{k_1^2}{2K''_{2\perp}}, \qquad K''_{2\perp} \approx \sqrt{|\boldsymbol{k}_\parallel|^2 - \epsilon' k_1^2}. \tag{11}$$

The '+' sign before imaginary part $iK''_{2\perp}$ determines exponential decay of the refracted wave away from the interface for both lossy and gainy media cases. However the real part, $K'_{2\perp}$ has opposite signs for lossy and gainy cases.

The reflection amplitudes (7) at TIR look

$$\rho_s = \frac{k_{1\perp} - iK''_{2\perp} \mp K'_{2\perp}}{k_{1\perp} + iK''_{2\perp} \pm K'_{2\perp}}, \tag{12}$$

$$\rho_p = \frac{\epsilon_2 k_{1\perp} - \epsilon_1 (iK''_{2\perp} \pm K'_{2\perp})}{\epsilon_2 k_{1\perp} + \epsilon_1 (iK''_{2\perp} \pm K'_{2\perp})}. \tag{13}$$

The positive value of $K'_{2\perp}$ for lossy medium means that the reflection coefficient in TIR is less than one, because part of the energy flux proportional to $K'_{2\perp}$ enters the medium 2 and is absorbed there. The negative value of $K'_{2\perp}$ for gainy medium means that the reflection coefficient in TIR is larger than one, because part of the energy flux proportional to $K'_{2\perp}$, exits the medium 2 and adds to the TIR wave.

In [2] it was incorrectly claimed (see, for example, critics in [3]) that in the case of TIR from a gainy medium the wave vector inside the gainy medium has opposite sign: $k_{2\perp} = K'_{2\perp} - iK''_{2\perp}$, and the reflection coefficient at TIR $|\rho_{s,p}|^2$ is less than unity. If it were so, then the wave function inside the gainy medium would increase proportionally to $\exp(K''_{2\perp}z)$ independently of how small is the gain. Since $K''_{2\perp} \sim 1/\lambda$ (see (11)) then for $\lambda \sim 1000$ nm the intensity of the field inside the gainy medium at a distance 1 mm from the interface would be larger than intensity of the incident wave of light by the factor $e^{2000} \approx 10^{860}$, which surpasses all the astronomical numbers. It proves that the claim in [2] can not be true.

With the correct sign $k_{2\perp} = -K'_{2\perp} + iK''_{2\perp}$ the reflection coefficient at TIR from a gainy medium is larger than one, and it increases with the gain. The photons induced by the incident wave cannot propagate inside the gainy medium by the same reason as the incident one. They can only go by the tunnelling transmission into the first medium. The increase in the reflected flux is due to the sub-barrier induction of the photon, which tunnels from the gainy medium into medium 1 and coherently adds to the reflected primary photon. The larger is the gain, the larger is the probability of such a process.

II. THE EXPERIMENT FOR STRONG ENHANCEMENT OF THE LIGHT TRAPPED IN A GLASS SPHERE

The increase of the reflection coefficient at TIR from a gainy medium can be used to develop a curious experiment for storage and amplification of light. Imagine a glass sphere with a coupler P, as shown in Fig.1. The sphere has thin walls (it is also possible to use a homogeneous glass sphere) and is surrounded by an excited gas (or other active media). The ray of light, shown by the thin line, enters the glass walls through the coupler and then undergoes TIR multiple times. At every reflection the light is amplified according to the analysis in the previous Section. It is possible to imagine a geometry in which the ray after entering the glass becomes trapped in it, or after sufficiently many reflections escapes the sphere, as shown by the thick line. The amplification depends on the number of the reflections and on the gain coefficient of the active medium. The number of the reflections is very sensitive to the angle of the incident ray. It is important to note, that the energy accumulates inside the sphere and does not go out of it. The total reflection works like a pump, and the pumped energy density can be much larger than the energy density in the surrounding gainy medium. If the overall amplification is sufficiently high, the glass will melt into a liquid bubble with thin skin filled with the light, similar to the ball lightning described in [4].

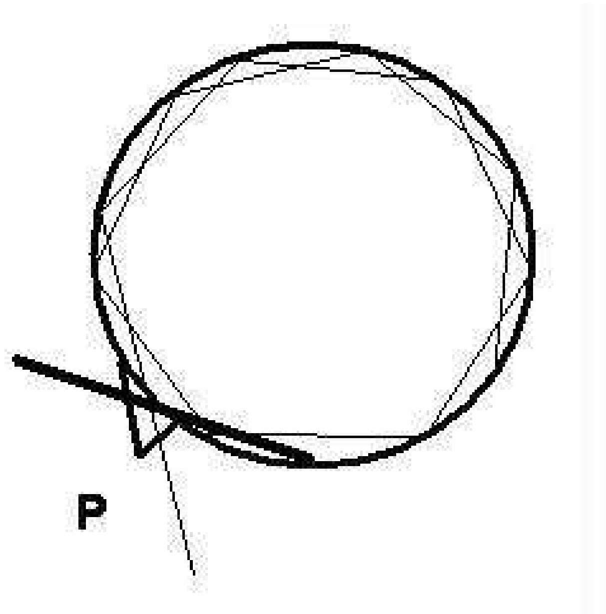

FIG. 1: Schematic of the experiment for multiple TIR off gainy medium.

We can estimate the magnitude of the light enhancement in such a sphere. Assume that for the active medium $\epsilon_2 \approx 1 - i\alpha$, and ϵ_1 of the glass is real. For TE-mode, the reflection amplitude at TIR according to Eq. 12 and 11 can be written as

$$\rho_s \approx \frac{k_{1\perp} - iK_{2\perp} + \alpha k_0^2/2K_{2\perp}}{k_{1\perp} + iK_{2\perp} - \alpha k_0^2/2K_{2\perp}}, \tag{14}$$

where $K_{2\perp} = \sqrt{(\epsilon_1 - 1)k_0^2 - k_{1\perp}^2}$, and the approximation is valid for small α. Similarly we can write equation for ρ_p. For estimating purposes we can assume that in both cases the light is amplified approximately by the factor $1 + 2\alpha$ at each reflection.

Let's consider a sphere of radius $R = 10$ cm submerged into an active medium with small α. The energy I inside it increases with number N of collisions with the walls $\propto (1+2\alpha)^N \approx \exp(2N\alpha)$. The N can be represented via time t as t/t_1, where $t_1 = 2R\sin\theta\sqrt{\epsilon_1}/c$ is the time between two consecutive collisions of photons with the walls, and θ is the grazing angle with surface at the collision point. Therefore $I(t) = I_0 \exp(t/\tau)$, where $\tau = t_1/2\alpha = R \sin\theta\sqrt{\epsilon_1}/c$, and I_0 is initial energy of the incident ray. For $\theta = 0.1$ and $\alpha = 10^{-7}$ we get $1/\tau \approx 3 \times 10^3$ s^{-1}. Therefore, for $I_0 = 10^{-19}$ J, the energy I after 20 ms reaches 10 MJ.

The following analysis is used to estimate α. The amplification of a laser beam along a path l inside a gainy media is $\exp(2k''l)$, where k'' is the imaginary part of the wave number, and $g = 2k''$ is called the gain coefficient. In a medium with $\epsilon = 1 - i\alpha$, the gain coefficient is $g \approx \alpha k = 2\pi\alpha/\lambda$, where λ is the wavelength. For N_2,CO_2 gas lasers, the gain coefficient is approximately 10^{-2} cm^{-1} [5]. For $\lambda/2\pi \simeq 10^{-4}$ cm we obtain $\alpha = 10^{-6}$.

In the past, many experiments were performed with the whispering gallery mode resonators (WGMR) of small dimensions (~ 1 mm) and large Q-factors (up to $Q \sim 10^{10}$) [6], where light undergoes large number $N \sim Q$ total internal reflections. In a larger sphere submerged into an active medium with $\alpha \sim 10^{-7}$, the Q-factor can also be large. Therefore we can accumulate there enormous energy. The stored photons will heat and melt the resonator, but the electrostriction forces will hold the melted substance together. One can expect to see many interesting nonlinear phenomena in such systems.

A. Derivation of the energy increase with the help of spherical harmonics

In [6, 7] and many other works with microspheres an analysis with spherical harmonics is used. It means that TE or TH field in and out of the sphere is represented by functions like

$$\psi(\boldsymbol{r}, t) = \exp(-i\omega t) Y_{l,m}(\theta, \phi) F_l(r), \tag{15}$$

where $Y_{l,m}(\theta, \phi)$ are the usual spherical harmonics, and the radial function $F_l(r)$ can be represented as

$$F_l(r) = \Theta(r < R)\frac{j_l(nk_0 r)}{j_l(nk_0 R)} + \Theta(r > R)\frac{h_l^{(1)}(n'k_0 r)}{h_l^{(1)}(n'k_0 R)}, \tag{16}$$

where $j_l(kr)$, $h_l^{(1)}(k'r)$ are spherical Bessel and Hankel functions respectively, n, n' are refraction indices inside and outside the sphere, $k_0 = \omega/c$ and the factor before $h_l^{(1)}(n'k_0 r)$ provides continuity of the function $F_l(r)$ at $r = R$.

The second boundary condition, requiring continuity of the radial derivative determines values of k_0 for which solution in the form (16) is possible.

However such an approach is not appropriate for trapped light, because outside function must be evanescent, while spherical Hankel functions are not. For description of the trapped light in WG mode, which is distributed closely to the sphere radius R and corresponds to $l \gg 1$, we can use expansion [8]

$$\frac{l^2}{r^2} \approx \frac{l^2}{R^2} - 2(r-R)\frac{l^2}{R^3}, \tag{17}$$

treat the linear term as a perturbation, and then in the simplest approximation the radial equation becomes

$$\left(\frac{d^2}{dr^2} + \epsilon k^2 - \frac{l^2}{R^2}\right) F_l(r) = 0. \tag{18}$$

Its solution is

$$F_l(r) = \Theta(r<R)\frac{\sin(k_r r)}{\sin(k_r R)} + \Theta(r>R)e^{-K_r(r-R)}, \tag{19}$$

where $k_r \approx \sqrt{\epsilon k_0^2 - l^2/R^2}$, $K_r = \sqrt{(\epsilon-\epsilon')k_0^2 - k_r^2}$. To get WGM we should have $\epsilon' < \epsilon$, and sufficiently large l for arguments of both square roots to be positive.

The second boundary condition will give limitations (or quantization) for k_0^2. If the medium outside the sphere is gainy one, then ϵ' contains negative imaginary part $-i\epsilon''$, and the second boundary condition will make k_0^2 or ω complex numbers with positive imaginary part $i\omega''$. It means that the factor $\exp(-i\omega t)$ in (15) provides exponential grows $\sim \exp(\omega'' t)$.

We do not follow this way, because approximation (18) is equivalent to reflection in plane geometry, and representation of wave function in the form (15) means that distribution of field in the sphere is periodic or all the rays in WG mode are closed. In general, it is not so like for the rays shown in fig.1. Therefore, if $Rk_0 \gg 1$ the analysis of trapped light in WG mode with spherical harmonics is not appropriate.

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