QUANTUM SCATTERING THEORIES, THEIR CONTRADICTIONS AND ATTEMPTS TO RESOLVE THEM

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Abstract

There are 3 quantum scattering theories: which we denote as spherical waves, Fermi golden rule, and fundamental theory. All of them have contradictions. We demonstrate contradictions and try to resolve them. A final conclusion is that noncontradictory theory requires wave packets, quantization of scattering angle and nonlinearity. There are no such a theory still, and we have nothing to do but to use the existing theories notwithstanding the contradictions inherent to them. Censorship against this paper is demonstrated.

1 Introduction

Quantum mechanics deals with dimensionless probabilities. However scattering data are presented in terms of cross sections. We will show that in fact scattering process is also described in terms of probabilities, and relation between cross section and probability is not straightforward one. A cross section can be obtained only in a nonlinear scattering theory. It is really a surprise that the scattering theory is so much successfully developed notwithstanding of its weak basement.

In fact we have three scattering theories: one operates with spherical waves (SWT). It is used mainly for description of elastic processes. The second one is the standard scattering theory (SST) based on the Fermi golden rule. It has much wider application. And the third theory is the fundamental one (FST), which provides an S-matrix in quantum field theory and is believed to prove validity of the SST.

Below we consider all the three theories, find their contradictions, try to resolve them and arrive to the new and new ones. Finally we find that there is no completely satisfactory scattering theory, and hope that this report about our endeavors [1] will be helpful to those who will try to construct it.

2 Spherical waves

To uncover the problem it is sufficient to consider the simplest case of s-wave elastic scattering. It is common to describe elastic scattering of low energy scalar particles on a fixed center (for example, slow neutrons on a heavy spinless nucleus) by the stationary wave function (see, for example [2] Ch. 10)

$$\psi(\boldsymbol{r}) = \exp(i\boldsymbol{k}\cdot\boldsymbol{r}) - b\frac{\exp(ikr)}{r}. \tag{1}$$

Here the first term is a plain wave of an incident particle, and the second term is the scattered spherical wave. It contains factor b called scattering amplitude, from which one obtains the scattering cross section $\sigma = 4\pi|b|^2$ with dimension of cm^2.

The wave function of the incident particle satisfies the free Schrödinger equation

$$(\Delta + k^2)\exp(i\boldsymbol{k}\cdot\boldsymbol{r}) = 0, \tag{2}$$

but the scattered wave does not satisfy the free Schrödinger equation. It satisfies the inhomogeneous equation

$$(\Delta+k^2)\frac{\exp(ikr)}{r}=-4\pi\delta(\boldsymbol{r}). \tag{3}$$

Therefore the spherical wave does not describe a free particle, while one expects that scattering of a free particle gives after scattering also a free particle described by a plain wave similar to that one of the incident particle. Therefore the wave function of the scattered particle should satisfy the homogeneous Eq. (2) rather than Eq. (3). However nobody worries about that because scattered particles are measured far away from the scattering point $r=0$, and there Eq. (3) has zero at the right hand side.

Such a consolation is nevertheless poor, which is demonstrated by the following example. Let's consider a bound state of a particle in a spherical potential well of finite width d. Its wave function satisfies the Schrödinger equation, which at $r>d$ coincides with the free one. However no one believes that the wave function of the bound state outside the potential well describes a free particle, because such a wave function have an exponentially decreasing asymptotics at $r\gg d$, while a free particle is described by a propagating exponent.

Let's accept exponential decay as a feature distinguishing non free particle from the free one, and find such an asymptotics of scattered wave function, which contains only propagating plain waves. The spherical wave is not an appropriate asymptotics, because besides the propagating plain waves it contains also exponentially decaying ones. To show it we represent $\exp(ikr)/r$ in the form of 2-dimensional Fourier expansion

$$\frac{\exp(ikr)}{r}=\frac{i}{2\pi}\int\exp(i\boldsymbol{p}_\parallel\boldsymbol{r}_\parallel+ip_zz)\frac{d^2p_\parallel}{p_z}, \tag{4}$$

where $p_z=\sqrt{k^2-p_\parallel^2}$, $z>0$ is the observation point and the integral runs over all $p_\parallel$. At $p_\parallel>k$ we have $p_z=i\sqrt{p_\parallel^2-k^2}$, so the function under the integral becomes exponentially decaying. To get rid of exponentially decaying waves we need to restrict integration in Eq. (4) to $p_\parallel<k$. In that case the integral in Eq. (4) after substitution

$$\frac{d^2p_\parallel}{p_z}=d^3p\delta(p^2/2-k^2/2)\Theta(p_zz>0), \tag{5}$$

transforms to

$$\psi_s(\boldsymbol{r})=\frac{i}{2\pi}\int\exp(i\boldsymbol{p}\boldsymbol{r})\Theta(p_zz>0)\,d^3p\delta(p^2/2-k^2/2)=\frac{ik}{2\pi}\int\limits_{4\pi}\exp(i\boldsymbol{k}_\Omega\boldsymbol{r})d\Omega, \tag{6}$$

where the wave vector $\boldsymbol{k}_\Omega$ has the length k and points to direction $\boldsymbol{\Omega}$. Therefore instead of Eq. (1) the correct wave function with account of scattering is

$$\psi(\boldsymbol{r})=\exp(i\boldsymbol{k}\cdot\boldsymbol{r})-\frac{ibk}{2\pi}\int\limits_{4\pi}\exp(i\boldsymbol{k}_\Omega\boldsymbol{r})d\Omega. \tag{7}$$

Such a wave function shows that the scattering produces a set of propagating plain waves isotropically distributed with probability amplitude $ibk/2\pi=b/\lambda$ in all the intervals of solid angles $d\Omega$. Therefore, the scattering from a single center is described by scattering probability

$$w_s(\boldsymbol{k}\to\boldsymbol{k}_\Omega)=(b/\lambda)^2d\Omega. \tag{8}$$

It looks nice, but then the question arises, how can we describe the data of the experiment schematically presented in Fig. 1? In it a large sample S with thickness d encloses all the

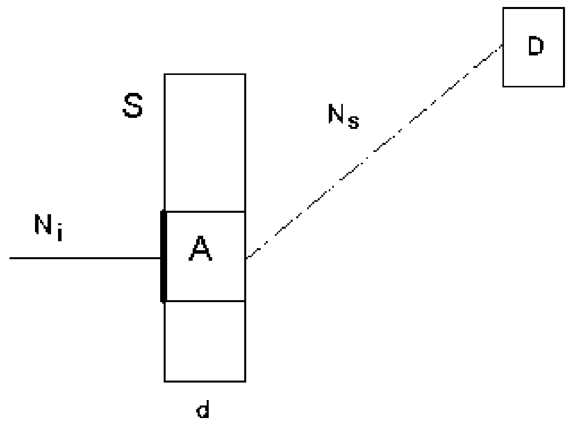

Figure 1: Definition of the cross section for scattering on a single atom

incident beam with flux N_i particles per sec. The scattered particles are counted by a detector D, and countrate is N_s. The only result of the experiment is the dimensionless ratio

$$W(\boldsymbol{k}_i \to \boldsymbol{k}_f) = \frac{N_s}{N_i}, \tag{9}$$

which gives probability of a particle scattering on its way through the whole sample. From this probability experimentalists derive scattering cross section per atom by simple division of the dimensionless probability W by dimensional number $N_0 d$, where N_0 is the atomic number density in the sample. So, this definition is a purely phenomenological one and it may have no relation to an individual process.

Logically we must find probability of scattering on a single atom. To find it we must know how many atoms N the particle meets on its way through the sample. To find N we have to suggest that the particle wave function has its own cross area denoted by A in Fig.1, i.e. it is a wave packet with space dimension $\sqrt{A}$. Then $N = N_0 A d$, and if the particle interacts only with these atoms (later we will see that such a suggestion can be wrong) then probability of the scattering on an individual atom is

$$w(\boldsymbol{k}_i \to \boldsymbol{k}_f) = \frac{W(\boldsymbol{k}_i \to \boldsymbol{k}_f)}{N_0 A d} = \frac{N_s}{N_i N_0 A d}, \tag{10}$$

and we can define the cross section

$$\sigma(\boldsymbol{k}_i \to \boldsymbol{k}_f) = A w(\boldsymbol{k}_i \to \boldsymbol{k}_f) = \frac{W(\boldsymbol{k}_i \to \boldsymbol{k}_f)}{N_0 d} = \frac{N_s}{N_i N_0 d}, \tag{11}$$

which coincides with the usual presentation of experimental data.

To compare a theory with an experiment we should be able to calculate probability of scattering $w(\boldsymbol{k}_i \to \boldsymbol{k}_f)$ and cross area A. We can calculate probability Eq. (8), if, for instance, interaction with a single scatterer is $4\pi b\delta(\boldsymbol{r})$, which gives the scattering wave function Eq. (7). However we have no idea how to calculate A. We can suggest that $A = \lambda^2$. Then we obtain the common expression

$$d\sigma(\boldsymbol{k}_i \to \boldsymbol{k}_f) = |b|^2 d\Omega, \tag{12}$$

but how can we justify this choice? In fact, we will see that $A \gg \lambda^2$. From this it follows that

$$d\sigma(\boldsymbol{k}_i \to \boldsymbol{k}_f) \gg |b|^2 d\Omega, \tag{13}$$

but it contradicts to experiment, if we accept the scattering amplitude at the level 10^{-12} cm. Does it not prove that all our objections against spherical waves are wrong?

We will analyze this point later after discussion of other theories. But before that let's note that we do not need cross section or probability for an individual atom in the case of coherent scattering on many atoms (to scatter simultaneously on many atoms A, should be large!), which leads to diffraction or reflection. There we really deal with dimensionless values and directly find the scattering amplitude b from Eq. (9). The problem starts when we try to describe incoherent processes and absorption.

3 The standard scattering theory (SST) with Fermi golden rule

The SST (see, for example [3,4]) in fact is not a theory but a list of recipes to cook a scattering cross section. The result is believed to be proven correct (we will show that this belief is incorrect) by the fundamental scattering theory [5,6].

The recipes of SST are:

1. Define the cross section as a ratio

$$d\sigma = \frac{1}{J_i} dW_F, \tag{14}$$

where J_i is the flux density of a single incident particle, and dW_F is probability of scattering of the single particle per unit time:

2. Define the probability of scattering per unit time according to the Fermi Golden Rule

$$dW_F(\boldsymbol{k}_i, \lambda_i \to \boldsymbol{k}_f, \lambda_f, t) = \frac{2\pi}{\hbar} \left|\langle \lambda_f, \boldsymbol{k}_f | V | \lambda_i, \boldsymbol{k}_i \rangle\right|^2 \rho(E_{fk}), \tag{15}$$

where $\langle \lambda_f, \boldsymbol{k}_f | V | \lambda_i, \boldsymbol{k}_i \rangle$ is a matrix element of the particle-scatterer interaction potential V between initial $|\boldsymbol{k}_i >$, $|\lambda_i >$, and final $|\boldsymbol{k}_f >$, $|\lambda_f >$, particle and scatterer states, respectively, and $\rho(E_{fk})$ is the density of the particle final states:

$$\rho(E_{fk}) = \delta(E_{ik} + E_{i\lambda} - E_{fk} + E_{f\lambda}) \frac{L^3 d^3 k_f}{(2\pi)^3}. \tag{16}$$

The delta-function factor corresponds to the energy conservation law. It contains initial and final particle $E_{i,fk}$ and scatterer $E_{i,f\lambda}$ energies. The last factor is the phase space density of the particle final states, which includes momentum element $d^3 k_f$ and some volume L^3 with an arbitrary large size L.

3. Define particle states before and after scattering as (they are really free states, comparing to spherical waves)

$$|\boldsymbol{k}_i >= L^{-3/2} \exp(i \boldsymbol{k}_{i,f} \boldsymbol{r}), \tag{17}$$

and with them the flux density of the single incident particle

$$J_i = \hbar \frac{k_i}{L^3}. \tag{18}$$

4. The ratio (14) gives the cross section

$$d\sigma(\boldsymbol{k}_i,\lambda_i\to\boldsymbol{k}_f,\lambda_f)=\frac{2\pi m}{\hbar^2 k_i}\left|\langle\lambda_f,\boldsymbol{k}_f|V|\lambda_i,\boldsymbol{k}_i\rangle\right|^2\delta(E_{ik}+E_{i\lambda}-E_{fk}+E_{f\lambda})\frac{L^6d^3k_f}{(2\pi)^3}. \quad (19)$$

Taking into account that $E_{fk}=\hbar^2k_f^2/2m$, $d^3k_f=mk_fdE_fd\Omega_f/\hbar^2$, one obtains double differential scattering cross section

$$\frac{d^2\sigma}{dE_fd\Omega_f}(\boldsymbol{k}_i,\lambda_i\to\boldsymbol{k}_f,\lambda_f)=\frac{m^2k_f}{\hbar^4k_i}\left|\langle\lambda_f,\boldsymbol{k}_f|V|\lambda_i,\boldsymbol{k}_i\rangle\right|^2\delta(E_{ik}+E_{i\lambda}-E_{fk}+E_{f\lambda})\frac{L^6}{(2\pi)^2}. \quad (20)$$

To compare with experimentally measured value one averages (20) over initial states and sums over final states of the scatterer, getting

$$\frac{d^2\sigma}{dE_fd\Omega_f}(\boldsymbol{k}_i\to\boldsymbol{k}_f,\mathcal{P})=$$

$$\sum_{\lambda_i,\lambda_f}\mathcal{P}(\lambda_i)\frac{m^2k_f}{\hbar^4k_i}\left|\langle\lambda_f,\boldsymbol{k}_f|V|\lambda_i,\boldsymbol{k}_i\rangle\right|^2\delta(E_{ik}+E_{i\lambda}-E_{fk}+E_{f\lambda})\frac{L^6}{(2\pi)^2}, \quad (21)$$

where $\mathcal{P}(\lambda_i)$ is the probability of scatterer to be initially in the state $|\lambda_i\rangle$.

One of contradictions here is that the probability of scattering per unit time (15) does not depend on time, so integral of dW_F over time is meaningless, whereas according to definition it should be less than unity.

Two other dubious points are: introduction of the superfluous volume L^3 and of flux density for a **single** particle. However in some respect introduction of L is like an introduction of a wave packet.

4 The Fundamental Scattering Theory (FST)

In FST (see, for example [5,6]) the scattering process is divided into three stages: infinite past, where an incident particle is free; present, where the particle interacts with a scatterer; and infinite future, where the scattered particle is again free. FST is used for derivation of S-matrix in quantum field theory and to prove validity of SST. First we outline the main ideas there and show the deficiency in the proof. After that we remind derivation of the S-matrix in field theory, show that it defines probability amplitude for transition between plain waves, like was found in SWT, and discuss the prescriptions used to get cross sections from probabilities.

4.1 The main ideas of FST

To get a rigorous proof it is necessary to work within the Hilbert space of normalized states. Therefore the incident free particle in FST is represented by a wave packet $|\phi\rangle$ and its Fourier expansion:

$$|\phi\rangle\equiv|\phi(\boldsymbol{r},\boldsymbol{k},s)\rangle=\int d^3pa(\boldsymbol{k}-\boldsymbol{p},s)|\boldsymbol{p}\rangle, \quad (22)$$

where parameter s characterizes dimension of the wave packet, $\boldsymbol{k}$ is momentum of the whole packet, $|\boldsymbol{p}\rangle$ is a state corresponding to the plane wave, $\exp(i\boldsymbol{pr})$, in coordinate space, and $a(\boldsymbol{p})$ are numerical Fourier coefficients. Dynamics of the packet $|\phi\rangle$ is determined by a free hamiltonian H_0:

$$|\phi(t)\rangle=\exp(-iH_0t)|\phi\rangle. \quad (23)$$

After scattering the wave function becomes

$$|\chi\rangle = \int d^3p \hat{S}|\boldsymbol{p}\rangle a(\boldsymbol{k}-\boldsymbol{p}, s), \tag{24}$$

where $\hat{S}$ is the scattering S-matrix. With the help of unity

$$I = \int d^3k' |\boldsymbol{k}'\rangle\langle\boldsymbol{k}'| \tag{25}$$

the scattered state Eq. (24) is represented as

$$|\chi\rangle = \int d^3k' |\boldsymbol{k}'\rangle \int d^3p \langle\boldsymbol{k}'|\hat{S}|\boldsymbol{p}\rangle a(\boldsymbol{k}-\boldsymbol{p}, s), \tag{26}$$

and dimensionless probability of scattering is defined as

$$dw(\boldsymbol{k}\to\boldsymbol{k}') = d^3k' |\langle\boldsymbol{k}'||\chi\rangle|^2 = d^3k' \left| \int d^3p \langle\boldsymbol{k}'|\hat{S}|\boldsymbol{p}\rangle a(\boldsymbol{k}-\boldsymbol{p}, s) \right|^2. \tag{27}$$

This last formula defines a probability for scattering of the wave packet into a final free particle state described by the plain wave $\exp(i\boldsymbol{k}'\boldsymbol{r})$. It means that scattering transforms the wave packet into a plain wave. Such a process completely violates unitarity. The state $|\phi\rangle$ is normalized and $|\boldsymbol{k}'\rangle$ is not normalizable. This is the main deficiency of FST.

4.2 Derivation of S-matrix in FST [5,6].

The incident state, described by the wave packet Eq. (22) is denoted as $|\psi_{\rm in}\rangle$. Its dynamics far away from the scatterer (target) is described by the free Hamiltonian H_0 as shown in Eq. (23).

The current state of the particle at the time $t \approx 0$, when the particle interacts with the target, is denoted as $|\Psi\rangle$. Its dynamics is described by the full Hamiltonian H, which contains interaction potential V:

$$|\Psi(t)\rangle = \exp(-iHt)|\Psi\rangle. \tag{28}$$

The state $|\Psi\rangle$ is related to $|\psi_{\rm in}\rangle$ via the limiting Möller operator [6] Ω_+, defined as

$$\Omega_+ = \lim_{t\to-\infty} U_+(0,t), \tag{29}$$

where

$$U_+(0,t) = e^{iHt}e^{-iH_0t}. \tag{30}$$

So

$$\Psi(0) = \Omega_+|\psi_{\rm in}\rangle. \tag{31}$$

The Möler operator depends on the interaction potential. From the equation

$$(d/dt)U_+(0,t) = ie^{iHt}Ve^{-iH_0t} \tag{32}$$

one finds

$$\Omega_+ = 1 - i \int\limits_{-\infty}^{0} e^{iHt'} V e^{-iH_0 t'}\, dt', \tag{33}$$

and it follows that

$$|\Psi(0)\rangle = \left(1 - i \int\limits_{-\infty}^{0} dt'\, e^{iHt'} V e^{-iH_0 t'}\right) |\phi\rangle = \int d^3p\, a(\boldsymbol{k}-\boldsymbol{p}, s) \left[1 - i \int\limits_{-\infty}^{0} dt'\, e^{i(H-E_p)t'} V\right] |\boldsymbol{p}\rangle, \tag{34}$$

where $a(\boldsymbol{k} - \boldsymbol{p}, s)$ are defined in Eq. (22), and we used the relation $\exp(-iH_0 t)|\boldsymbol{p}\rangle = \exp(-iE_p t)|\boldsymbol{p}\rangle$. Integration over t' in (34) leads to

$$|\Psi(0)\rangle = \int d^3p\, a(\boldsymbol{k} - \boldsymbol{p}, s) \left[1 - \frac{1}{H - E_p - i\epsilon} V\right] |\boldsymbol{p}\rangle = \int d^3p\, a(\boldsymbol{k} - \boldsymbol{p}, s) |\psi_{\boldsymbol{p}}\rangle. \tag{35}$$

where functions $|\psi_{\boldsymbol{p}}\rangle$ replace plane waves $|\boldsymbol{p}\rangle$ in the packet.

The intermediate function, which in the wave packet expansion Eq. (35) replaces plane waves, is

$$|\psi_{\boldsymbol{p}}\rangle = \left[1 - \frac{1}{H - E_p - i\epsilon} V\right] |\boldsymbol{p}\rangle. \tag{36}$$

It satisfies the stationary Schrödinger equation with full Hamiltonian: $(H - E_p)|\psi_{\boldsymbol{p}}\rangle = 0$, and, according to the standard theory, contains incident plain and scattered spherical waves.

After scattering at an infinite future the state of the free particle is denoted $|\psi_{\rm out}\rangle$. Its dynamics is governed again by the free Hamiltonian H_0:

$$|\psi_{\rm out}(t)\rangle = \exp(-iH_0 t)|\psi_{\rm out}\rangle. \tag{37}$$

This state is also a wave packet of the type $|\psi_{\rm out}\rangle = \int d^3k'\, |\boldsymbol{k}'\rangle b(\boldsymbol{k}, \boldsymbol{k}', s)$.

The state $|\psi_{\rm out}\rangle$ is asymptotically related to $|\Psi\rangle$: $\exp(-iHt)|\Psi\rangle \to \exp(-iH_0 t)|\psi_{\rm out}\rangle$, from which the second Möler operator is defined

$$\Omega_- = \lim_{t\to\infty} U_-(0, t), \tag{38}$$

where

$$U_-(0, t) = e^{iHt} e^{-iH_0 t}. \tag{39}$$

Therefore $\Psi(0) = \Omega_- |\psi_{\rm out}\rangle$ is established. From Eq. (35) it follows that

$$|\psi_{\rm out}\rangle = \Omega_-^{-1}\Psi(0) = \lim_{t\to\infty} e^{iH_0 t} e^{-iHt} \int d^3p\, a(\boldsymbol{k}-\boldsymbol{p}, s)|\psi_{\boldsymbol{p}}\rangle = \lim_{t\to\infty} e^{iH_0 t} \int d^3p\, a(\boldsymbol{k}-\boldsymbol{p}, s) e^{-iE_p t}|\psi_{\boldsymbol{p}}\rangle. \tag{40}$$

Substitution of unity $I = \int |\boldsymbol{k}'\rangle d^3k' \langle \boldsymbol{k}'|$ after $\exp(-iH_0 t)$ gives

$$|\psi_{\rm out}\rangle = \lim_{t\to\infty} \int d^3p\, a(\boldsymbol{k} - \boldsymbol{p}, s) \int d^3k'\, e^{iE_{k'} t} |\boldsymbol{k}'\rangle \langle \boldsymbol{k}'||\psi_{\boldsymbol{p}}\rangle e^{-iE_p t}. \tag{41}$$

Relation between two asymptotical states is established by substitution of (36) into (41)

$$|\psi_{\rm out}\rangle = \lim_{t\to\infty} \int d^3k' |\boldsymbol{k}'\rangle \int d^3p\, a(\boldsymbol{k} - \boldsymbol{p}, s) \left[\delta(\boldsymbol{k}' - \boldsymbol{p}) - \langle \boldsymbol{k}'| \left(\frac{e^{i(E_{k'} - E_p)t}}{H - E_p - i\epsilon} V\right) |\boldsymbol{p}\rangle\right]. \tag{42}$$

A matrix $\mathcal{T}$ is introduced. It is defined with the help of the simple relation:

$$\frac{1}{H-E_p-i\epsilon}V=\frac{1}{H_0-E_p-i\epsilon}\left(1-V\frac{1}{H-E_p-i\epsilon}\right)V=\frac{1}{H_0-E_p-i\epsilon}\mathcal{T}. \tag{43}$$

Substitution of (43) into (42) gives

$$|\psi_{\rm out}\rangle=\lim_{t\to\infty}\int d^3k'|\boldsymbol{k}'\rangle\int d^3p\,a(\boldsymbol{k}-\boldsymbol{p},s)\left[\delta(\boldsymbol{k}'-\boldsymbol{p})-\frac{e^{i(E_{k'}-E_p)t}}{E_{k'}-E_p-i\epsilon}\langle\boldsymbol{k}'|\mathcal{T}|\boldsymbol{p}\rangle\right]. \tag{44}$$

At $t\to\infty$ we can use the relation

$$\lim_{\epsilon\to0}\frac{\exp(ixt)}{x-i\epsilon}\equiv i\int\limits_{-\infty}^{t}\exp(ixt')\,dt', \tag{45}$$

which transforms (44) to

$$|\psi_{\rm out}\rangle=\int d^3k'\,|\boldsymbol{k}'\rangle\int d^3p\,\langle\boldsymbol{k}'|\hat{S}|\boldsymbol{p}\rangle a(\boldsymbol{k}-\boldsymbol{p},s), \tag{46}$$

where $\hat{S}$-matrix is defined via its matrix elements

$$\langle\boldsymbol{k}'|\hat{S}|\boldsymbol{p}\rangle=\delta(\boldsymbol{k}'-\boldsymbol{p})-2\pi i\delta(E_{k'}-E_p)T(\boldsymbol{k}',\boldsymbol{p}), \tag{47}$$

with $T(\boldsymbol{k}',\boldsymbol{p})=\langle\boldsymbol{k}'|\mathcal{T}|\boldsymbol{p}\rangle$. This is the well known Lippmann-Schwinger equation [7, 8], which gives probability amplitudes for transition from a plain wave state with momentum $\boldsymbol{p}$ to the plain wave states with momenta $\boldsymbol{k}'$ in the phase space volume d^3k'.

It is important to note that if Fourier coefficient in expansion Eq. (22) is $a(\boldsymbol{k}-\boldsymbol{p},s)=\delta(\boldsymbol{k}-\boldsymbol{p})$, then substitution of Eq. (47) into Eq. (26) gives the wave function after scattering similar to Eq. (7). With such an approach it is possible to construct scattering theory [9] without artificial size parameter L.

Let's note that in the whole formalism described above, there are no dependence of matrix elements on position of target with respect to the wave packet. We must remember it in discussion of the tricks used in books by [5, 6] to get cross sections from probabilities.

4.3 Transition from Probabilities to Cross Sections

Now we show how the probability is transformed to a cross section in [5,6], but start with steps related to transformations of probability (27) that are common to both books.

According to (27), Eq. (44) and Eq. (47) the scattering probability ($\boldsymbol{k}'\neq\boldsymbol{p}$) is

$$dw(\boldsymbol{k}\to\boldsymbol{k}')=d^3k'\,(2\pi)^2T(\boldsymbol{k}',\boldsymbol{p})T^*(\boldsymbol{k}',\boldsymbol{p}')a(\boldsymbol{k}-\boldsymbol{p},s)a^*(\boldsymbol{k}-\boldsymbol{p}',s)\,d^3p\,d^3p'\,\delta(k'^2/2-p'^2/2)\,\delta(k'^2/2-p^2/2). \tag{48}$$

The spectrum of $\boldsymbol{p}$ in the wave packet is supposed to be sufficiently narrow around $\boldsymbol{k}$ so $\boldsymbol{p}$ and $\boldsymbol{p}'$ in the matrix elements $T(\boldsymbol{k}',\boldsymbol{p}')$ and $T(\boldsymbol{k}',\boldsymbol{p})$ can be replaced by $\boldsymbol{k}$. Therefore

$$T(\boldsymbol{k}',\boldsymbol{p})T^*(\boldsymbol{k}',\boldsymbol{p}')\approx|T(\boldsymbol{k}',\boldsymbol{k})|^2. \tag{49}$$

The product $\delta(k'^2/2-p'^2/2)\,\delta(k'^2/2-p^2/2)$ in (48) is identical to $\delta(p^2/2-p'^2/2)\,\delta(k'^2/2-p^2/2)$. Momentum p in $\delta(k'^2/2-p^2/2)$ is approximated by k, and $d^3k'\,\delta(k'^2/2-k^2/2)$ is replaced with $kd\Omega'$, where Ω' is the scattering angle.

The factor $\delta(p^2/2 - p'^2/2)$ is represented as

$$\delta(p^2/2 - p'^2/2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dt \exp\Big(it(p^2 - p'^2)/2\Big). \tag{50}$$

The difference in the exponent is approximated with $p^2 - p'^2 = (\boldsymbol{p}+\boldsymbol{p}')\cdot(\boldsymbol{p}-\boldsymbol{p}') \approx 2\boldsymbol{k}\cdot(\boldsymbol{p}-\boldsymbol{p}')$, and as a result the product $a(\boldsymbol{k}-\boldsymbol{p},s)a^*(\boldsymbol{k}-\boldsymbol{p}',s)\,d^3p\,d^3p'\,\delta(p^2/2 - p'^2/2)$ is reduced to

$$\int d^3p\,a(\boldsymbol{k}-\boldsymbol{p},s)\exp(i\boldsymbol{k}\cdot\boldsymbol{p}t)\int d^3p'\,a^*(\boldsymbol{k}-\boldsymbol{p}',s)\exp(-i\boldsymbol{k}\cdot\boldsymbol{p}'t) = |\phi(\boldsymbol{r},\boldsymbol{k},s)_{\boldsymbol{r}=\boldsymbol{k}t}|^2, \tag{51}$$

where the Fourier expansion (22) of the wave packet is used. If we direct z-axis along $\boldsymbol{k}$ and change integration variable dt to $dz = kdt$ then (48) is reduced to

$$dw = d\Omega'\,2\pi|T(\boldsymbol{k}',\boldsymbol{k})|^2\int_{-\infty}^{\infty} dz\,|\phi(z,k,s)|^2. \tag{52}$$

4.3.1 Transition to Cross Section According to [5]

Since the wave packet is normalized to unity, then $\int|\phi(z,k,s)|^2\,dz$ has dimensionality $1/\text{cm}^2$. It can be interpreted as a density of incident particles per unit area. So the ratio of probability (52) to density of incident particles is the cross section:

$$d\sigma = \frac{dw}{\int|\phi(z,k,s)|^2\,dz} = d\Omega'\,2\pi|T(\boldsymbol{k}',\boldsymbol{k})|^2. \tag{53}$$

4.3.2 Transition to Cross Section According to [6].

In [5] the wave packet is $\int d^3pa(\boldsymbol{p}-\boldsymbol{k})\exp(i\boldsymbol{p}\cdot\boldsymbol{r} - ip^2t/2)$, and the scatterer is supposed at the point $\boldsymbol{r} = 0$ as shown in fig.2a. Therefore the center of the wave packet at $t = 0$ coincides with the scatterer. In [6] the wave packet is chosen to be $\int d^3pa(\boldsymbol{p}-\boldsymbol{k})\exp(i\boldsymbol{p}\cdot(\boldsymbol{r}-\boldsymbol{\rho}) - ip^2t/2)$, and the scatterer again is supposed at the point $\boldsymbol{r} = 0$ as shown in fig.2b, therefore at $t = 0$ the packet is at the impact parameter ρ to the scatterer and J. Taylor instead of (52) obtained the probability

$$dw(\boldsymbol{\rho}) = d\Omega'\,2\pi|T(\boldsymbol{k}',\boldsymbol{k})|^2\int_{-\infty}^{\infty} dz\,|\phi(\boldsymbol{\rho},z)|^2. \tag{54}$$

The cross section is obtained by integration over impact parameter

$$d\sigma = \int d^2\rho\,dw(\boldsymbol{\rho}) = d\Omega'\,2\pi|T(\boldsymbol{k}',\boldsymbol{k})|^2\int_{-\infty}^{\infty} d^3r\,|\phi(\boldsymbol{r})|^2 = d\Omega'\,2\pi|T(\boldsymbol{k}',\boldsymbol{k})|^2, \tag{55}$$

where normalization of the incident wave packet was taken into account.

4.3.3 Analysis of Procedures by [5, 6]

The approximation (49) is equivalent to assumption that scattering of the wave packet is identical to scattering of the plane wave with momentum $\boldsymbol{k}$ into plane wave with momentum $\boldsymbol{k}'$. The division of probability by $|\phi(0,0,z)|^2$ in (53) or integration over impact parameter in (55) is absolutely equivalent to multiplication of the probability by the packet's cross area, i.e. $d\sigma = Adw$, as was deduced from experiment outlined in fig. 1. The considerations about position of the wave packet with respect to scatterer are absolutely not related to scattering itself. It only shows that they assume that there is no scattering, if the wave packet does not overlap the scatterer. This assumption is in accord with the common sense, and above we also followed it. However the scattering amplitude does not depend on any trajectory so the implicit assumption is not verified, and in the next section it is shown to be even wrong.

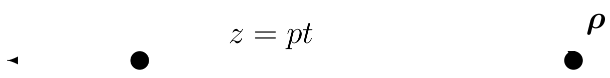

Figure 2: Position of the scatterer at $t = 0$ with respect to the center of the wave packet of the incident particle. On the left side they coincide. This is used in derivation of (53). On the right side the target is at the impact parameter $\boldsymbol{\rho}$ from the packet center. This geometry is used in derivation of (55).

5 Scattering of wave packets from a fixed center

In general all the wave packets can be represented by their Fourier expansion

$$\psi(\boldsymbol{r}, \boldsymbol{k}, t, s) = G(s|\boldsymbol{r} - \boldsymbol{k}t|)e^{i\boldsymbol{kr}-i\omega(k,s)t} = \int d^3p\, a(\boldsymbol{p}, \boldsymbol{k}, s)e^{i\boldsymbol{pr}-i\omega(\boldsymbol{p},\boldsymbol{k},s)t}, \tag{56}$$

where parameter s determines the width of the packet, $a(\boldsymbol{p}, \boldsymbol{k}, s)$ and $\omega(\boldsymbol{p}, \boldsymbol{k}, s)$ are functions of invariant variables $\boldsymbol{k}^2$, $\boldsymbol{p}^2$ and $\boldsymbol{kp}$.

The primary wave packet describes a free incident particle. Its Fourier expansion contains plane waves $\exp(i\boldsymbol{pr})$, which satisfy the equation

$$[\Delta + p^2]\exp(i\boldsymbol{pr}) = 0. \tag{57}$$

All the packets are representable in the form (56), and they differ by the Fourier coefficients $a(\boldsymbol{p}, \boldsymbol{k}, s)$ and dispersion $\omega(\boldsymbol{p}, \boldsymbol{k}, s)$. Very famous is the Gaussian wave packet, but we prefer de Broglie's singular non spreading normalized wave packet [10]

$$\psi_{dB}(\boldsymbol{r}, \boldsymbol{k}, t, s) = C\exp(i\boldsymbol{kr} - i\omega(k, s)t)\frac{\exp(-s|\boldsymbol{r} - \boldsymbol{v}t|)}{|\boldsymbol{r} - \boldsymbol{v}t|}, \tag{58}$$

where C is normalization constant: $C = \sqrt{s/2\pi}$. The parameter s is the width of the packet in momentum space and reciprocal width in coordinate space, $\boldsymbol{v} \equiv \boldsymbol{k}$ is the packet speed, and $\omega(k, s) = (k^2 - s^2)/2$. We see that $\omega(k, s)$ is less than kinetic energy by the term $s^2/2$, which can be thought of as bound energy of the packet.

The singular de Broglie wave packet satisfies inhomogeneous Schrödinger equation

$$\left[i\frac{\partial}{\partial t} + \frac{\Delta}{2}\right]\psi_{dB}(\boldsymbol{r}, \boldsymbol{v}, t, s) = -2\pi Ce^{i(v^2+s^2)t/2}\delta(\boldsymbol{r} - \boldsymbol{v}t), \tag{59}$$

with right hand side being zero everywhere except one point.

The Fourier coefficients of the singular de Broglie wave packet are

$$a_{dB}(\boldsymbol{p},\boldsymbol{k},s)=\sqrt{\frac{s}{2\pi}}\frac{4\pi}{(2\pi)^3}\frac{1}{(\boldsymbol{p}-\boldsymbol{k})^2+s^2}. \tag{60}$$

and

$$\omega_{dB}(\boldsymbol{p},\boldsymbol{k},s)=[2\boldsymbol{k}\boldsymbol{p}-k^2+s^2]/2=[p^2-(\boldsymbol{k}-\boldsymbol{p})^2-s^2]/2. \tag{61}$$

The spectrum of wave vectors $\boldsymbol{p}$ is spherically symmetrical with respect to the central point $\boldsymbol{p}=\boldsymbol{k}$ and decays away from it according to Lorenzian law with width s.

The Fourier coefficients (60) and frequency (61) become identical to those of spherical wave

$$\exp(-ik^2t/2)\frac{\exp(ikr)}{r}=\frac{4\pi}{(2\pi)^3}\int\exp(i\boldsymbol{p}\boldsymbol{r})\frac{\exp(-ik^2t/2)\,d^3p}{p^2-k^2-i\epsilon}, \tag{62}$$

after substitution $\boldsymbol{k}\to 0$ and $s\to -ik$, i.e. $\psi_{dB}(\boldsymbol{r},0,t,-ik)=C\exp(ikr-ik^2t/2)/r$.

The cross area of the singular de Broglie's wave packet moving in z direction can be defined as

$$A_{dB}=\int\pi\rho^2d^3r|\psi_{dB}(\boldsymbol{r},\boldsymbol{v},t,s)|^2=\frac{s}{2\pi}\int\limits_0^\infty 2dz\pi d\rho^2\pi\rho^2\frac{\exp(-2s\sqrt{\rho^2+z^2})}{\rho^2+z^2}=\frac{\pi}{3s^2}, \tag{63}$$

where $\rho=\sqrt{x^2+y^2}$.

Now we look at a wave packet elastic scattering from a fixed center. We take the wave packet not as a preparation of a particle in some state, but as an intimate property of the particle, which means that after scattering the particle is the same packet as before it.

The wave packet (56) relates to a free particle. In the presence of a potential $u(\boldsymbol{r})$ the plane wave components $\exp(i\boldsymbol{p}\boldsymbol{r})$ should be replaced by the wave functions $\psi_{\boldsymbol{p}}(\boldsymbol{r})$, which are solutions of the equation

$$[\Delta+p^2-u(\boldsymbol{r})]\psi_p(\boldsymbol{r})=0 \tag{64}$$

containing $\exp(i\boldsymbol{p}\boldsymbol{r})$ as the incident wave. Substitution of $\psi_p(\boldsymbol{r})$ into (56) transforms it to

$$\psi(\boldsymbol{r},\boldsymbol{k},t,s)=\int d^3p\,a(\boldsymbol{p},\boldsymbol{k},s)\psi_p(\boldsymbol{r})\exp[-i\omega(\boldsymbol{p},\boldsymbol{k},s)t]. \tag{65}$$

Now we have to find asymptotical form of (65). For that we substitute asymptotical form of $\psi_p(\boldsymbol{r})$. For incident wave $\exp(i\boldsymbol{p}\boldsymbol{r})$ asymptotical wave function after scattering on a fixed center with an impact parameter $\boldsymbol{\rho}$ is a superposition of plane waves:

$$\psi_p(\boldsymbol{r})\Rightarrow\exp(i\boldsymbol{p}\boldsymbol{\rho})\int d\Omega F'(p,\Omega)\exp(i\boldsymbol{p}_\Omega[\boldsymbol{r}-\boldsymbol{\rho}]), \tag{66}$$

where $F'(p,\Omega)$ is the probability amplitude of a plane wave with wave vector $\boldsymbol{p}$ to be transformed to the plane wave with wave vector $\boldsymbol{p}_\Omega$ pointing into direction $\boldsymbol{\Omega}$ in the element of solid angle $d\Omega$. This amplitude for isotropic scattering is $F'(p,\Omega)=bp/2\pi$. Dependence on p is an irritating moment, however, since the spectrum of wave packets has a sharp peak at $p=k$, we can approximate $F'(p,\Omega)$ by $bk/2\pi$, having in mind that corrections to this value is of the order s/k, where s is the packet width in the momentum space. This correction is small, when s is small, i.e. area A of the packet is large.

The vector $\boldsymbol{p}_\Omega$ in (66) is of length p, but it is turned by angle Ω from $\boldsymbol{p}$. Substitution of (66) into (56) for $\exp(i\boldsymbol{pr})$ transforms (56) to the form

$$\psi(\boldsymbol{r},\boldsymbol{k},t,s)=\int d^3p\,a(\boldsymbol{p},\boldsymbol{k},s)\exp(i\boldsymbol{p}\boldsymbol{\rho})d\Omega F'(k,\Omega)\exp[i\boldsymbol{p}_\Omega[\boldsymbol{r}-\boldsymbol{\rho}]-i\omega(\boldsymbol{p},\boldsymbol{k},s)t]. \tag{67}$$

Since $a(\boldsymbol{p},\boldsymbol{k},s)$, $\boldsymbol{p}\boldsymbol{\rho}$ and $\omega(\boldsymbol{p},\boldsymbol{k},s)$ are invariant with respect to rotation, we can replace them with $a(\boldsymbol{p}_\Omega,\boldsymbol{k}_\Omega)$, $\boldsymbol{p}_\Omega\boldsymbol{\rho}_\Omega$ and $\omega(\boldsymbol{p}_\Omega,\boldsymbol{k}_\Omega,s)$. After that we can transform integration variable $\boldsymbol{p}\to\boldsymbol{p}_\Omega$, and drop the index Ω of $\boldsymbol{p}$. As a result we transform (67) to the form

$$\psi(\boldsymbol{r},\boldsymbol{k},t,s)=\int d^3p\,a(\boldsymbol{p},\boldsymbol{k}_\Omega,s)\exp(i\boldsymbol{p}\boldsymbol{\rho}_\Omega)d\Omega F'(k,\Omega)\exp[i\boldsymbol{p}[\boldsymbol{r}-\boldsymbol{\rho}]-i\omega(\boldsymbol{p},\boldsymbol{k}_\Omega,s)t], \tag{68}$$

which can be represented as

$$\psi(\boldsymbol{r},\boldsymbol{k},t,s)=\int d\Omega F'(k,\Omega)\psi_0(\boldsymbol{r}-\boldsymbol{\rho}+\boldsymbol{\rho}_\Omega,\boldsymbol{k}_\Omega,t,s), \tag{69}$$

where ψ_0 denotes the wave packet of the the same form as that of the incident particle.

We see that the packet as a whole is scattered with probability

$$dw(\boldsymbol{k}\to\boldsymbol{k}_\Omega)=|F'(k,\Omega)|^2d\Omega=|bk/2\pi|d\Omega,$$

which, surprisingly, has no dependence on impact parameter $\boldsymbol{\rho}$ as in the case of plane waves. It shows that scattering of wide wave packets ($s\ll k$, $1/s\gg\lambda$) is almost the same as that of plane waves.

The independence of scattering amplitude on position of scatterer is well understandable in linear wave mechanics. Indeed, the wave packet is a superposition of plane waves, which exist in the whole space. They cancel each other outside the packet, but they exist, and because they are scattered independently of each other, the whole packet's scattering does not depend on position of the scatterer.

To get a cross section we need an additional hypothesis which restricts scattering to those cases, when the wave packet overlaps the target position. This hypothesis is outside of the wave mechanics and in the books [5, 6] it is accepted implicitly. We can say that without this hypothesis the wave mechanics is incomplete theory, i.e. it is insufficient to describe scattering of particles. Introduction of this hypothesis is equivalent to inclusion of nonlinearity into quantum mechanics. When we write $\sigma=Aw_1$, we implicitly accept nonlinearity.

6 Experimental investigation of the wave packet properties

We cannot deduce A, but we can explore its properties. So we can ask ourselves how large is A and what is its dependence on E.

6.1 Estimation of size of A

At first sight A should not be large. Indeed, if cross section of scattering from a fixed center, or from a heavy nucleus is $4\pi A|b|^2k^2/(2\pi)^2$, then, to get usual value $4\pi|b|^2$, we must put $A=(2\pi/k)^2=\lambda^2$, where λ is wave length of the particle motion with respect to the scattering center.

However, if A is small, then, it becomes impossible, for instance, total reflection of thermal neutrons from matter. Indeed, the wave packet (56) contains plain waves with different wave vectors $\boldsymbol{p}$. Probability of wave packet reflection from a matter is determined by the reflected wave function

$$\psi_r(\boldsymbol{r},\boldsymbol{k},t,s)=\int d^3p\,R(p_\perp)a(\boldsymbol{p},\boldsymbol{k},s)\exp[i\boldsymbol{pr}-i\omega(\boldsymbol{p},\boldsymbol{k},s)t], \tag{70}$$

where

$$R(p_\perp)=\frac{p_\perp-\sqrt{p_\perp^2-u}}{p_\perp+\sqrt{p_\perp^2-u}} \tag{71}$$

is reflection amplitude of the plain wave $\exp(i\boldsymbol{pr})$,

$$u=4\pi N_0 b \tag{72}$$

is interaction potential of neutron with matter, N_0 is atomic density, and $b\sim 10^{-12}$ cm is coherent neutron-atom scattering amplitude. The total reflection takes place when $p_\perp^2<u$ because in such a case the reflection amplitude $R(p_\perp)=\exp(i\phi)$ is modulo unity. The full wave packet is almost totally reflected if totally reflected almost all its constituent plain waves.

The width of the spectrum of $\boldsymbol{p}$ in the wave packet is proportional to s. Since $A\propto 1/s^2$, then if A is small, then s is large. If s is large, the number of waves with $p_\perp^2>u$ when $k_\perp^2<u$ is also large, so $\int d^3r|\psi_r(\boldsymbol{r},\boldsymbol{k},t\to\infty,s)|^2<1$. To get large reflection we need small s, because at $k_\perp^2<u$ the number of plain waves with $p_\perp^2>u$ is small. In that case reflection is almost total, i.e. $\mu=1-\int d^3r|\psi_r(\boldsymbol{r},\boldsymbol{k},t\to\infty,s)|^2\propto s\ll 1$. In physics of ultracold neutrons [11] (UCN) (neutrons with $k^2<u$) it is experimentally found that $\mu_{exp}\sim 3\cdot 10^{-5}$, which is much larger than theoretically estimated value $\mu_{th}\sim 3\cdot 10^{-7}$. If we attribute μ_{exp} to wave packet nature of the neutron [12] we find $s\approx 3\cdot 10^{-5}k$.

However, if s is small, then $A\propto 1/s^2$ is large. Therefore the cross section is

$$4\pi\frac{A|b|^2k^2}{(2\pi)^2}=4\pi\frac{|b|^2}{9(2\pi)^2}\cdot 10^{10}.$$

If we want it to have a value of the order of 1 barn as is usually accepted, we need to take b of the order of 10^{-16} cm. With such a small b we obtain the potential Eq. (72) too small. With such a potential no total reflection of thermal neutrons and storage of UCN are possible. So we arrive at a contradiction: the value of A have to be large, but it cannot be large. Later we will see how this contradiction can be resolved.

6.2 Some experiments to explore the wave-packet nature of particles

Here we will describe two types of experiments to explore wave packet nature of particles. Both of them are related to total or almost total reflection of neutrons from mirrors. However first of all we want to notify readers in advance that we consider wave packet as an immanent property of particles contrary to usual definition as preparation of an initial state. As an immanent property the wave packet should not spread. Therefore the Gaussian wave packet is not an appropriate one though this can be checked experimentally. Indeed, since the gaussian packet spreads then cross sections measured at small distances from a source should be smaller than cross sections measured far from the source. Such an experiment was never done.

We suppose that the best candidate for particles is the singular de Broglie's wave packet Eq. (58). It satisfies Eq. (59), i.e it can be interpreted as a field of the free moving particle. Effects which should be observed in described below experiments are proportional to width s of the wave packet.

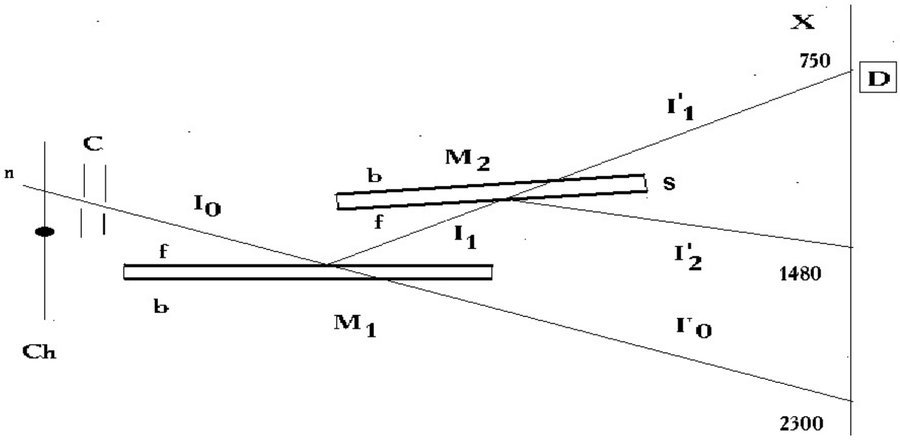

Figure 3: Scheme of the experiment [14] on search of subcritical transmission. After the chopper (Ch) and collimator (C) neutron beam (I_0) is specularly reflected (I_1) from the front surface (f) of the first Si mirror (M_1) and is specularly reflected (I_2) from the front surface (f) of the second mirror (M_2). M_2 is slightly tilted with respect to M_1. The grazing angle of I_0 with respect to M_1 is less than critical, and the grazing angle between I_1 and M_2 is even smaller. Detector (D) can be shifted along line X. Subcritical transmission (I'_1) was measured at the position $X = 750 \div 800$. Direct beam I_0 and double reflected beam I_2 were registered at positions $X = 2300$ and $X = 1480$ respectively, where $\Delta X = 100$ corresponds to shift equal 2 mm.

6.2.1 Subcritical transmission of thin plates

One type of experiments for investigation of wave packet is a search for subcritical transmission of thin plates. Such an experiment was reported in [13, 14]. Scheme of the experiment [14] is shown in fig.3. It was supposed that when a wave packet with speed $\boldsymbol{k}$ is totally reflected from the plate, some fraction of its plain wave constituents $\exp(i\boldsymbol{pr})$ have $p_\perp^2 > u$, so their reflection amplitude Eq. (71) is smaller than unity. Fraction of such waves determine nontunneling transmission of wave packet through the plate. In linear quantum mechanics such process means filtration. So the transmitted wave packet has higher energy, and reflected one has smaller energy, and both wave packets differ from the incident one. We suppose that reflected and transmitted wave packets remain the same, and fraction of waves with high $p_\perp$ determines only the probability of nontunneling transmission [13] $w \sim s/\sqrt{u}$. We also supposed that $s \propto k$, so the higher is energy of the incident neutrons, the narrower is their wave packet, and the higher is subcritical transmission.

The result of the experiment [14] was negative, so it was concluded that the width s does not depend on the neutron speed k. This conclusion was later checked in [15]. If wave packet width s does not depend on neutron energy, then transmission of cold neutrons through an atomic gas like He^4 at temperature T should show decrease of intensity $\propto T^{3/2}$. On the other side, if $s \propto k$, where k is the neutron speed, then transmission should show decrease of intensity $\propto T^{1/2}$. The experimental data [15] clearly favors $T^{1/2}$, therefore $s \propto k$, and previous experiments [14] must be repeated with higher accuracy.

6.2.2 Goos-Hänchen effect [16]

Scheme of another possible experiment is shown in fig.4. The goal is [16] to seek for deviation of reflection direction from the specular one. The deviation can be characterized by the angle $\gamma \sim s^2/kk'_\perp$ of rotation away from the external surface normal, where $k'_\perp = \sqrt{k_\perp^2 - u}$. This deviation can be of order of arc seconds, so it can be measured only with single crystals.

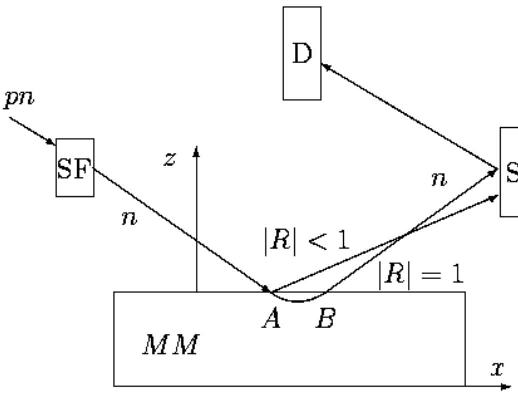

Figure 4: Scheme of experiment to check deviation of reflected neutrons from specular direction when reflection is a little bit above the critical one. Monochromatic polarized neutrons (pn) go through spin flipper (SF) and are reflected from a magnetized mirror (MM). If SF is switched off then neutron polarization is parallel to the mirror magnetization, reflection is total and specular. The neutrons reflected from MM go to Si single crystal and after Bragg reflection to detector D. When SF is switched on, neutron polarization becomes opposite to magnetization of MM, reflection becomes a little bit overcritical, and direction of the reflected neutron becomes nonspecular. Thus the Bragg condition for neutrons going to Si crystal is not satisfied. In order to restore Bragg condition the Si crystal must be turned by an angle γ. The purpose of the experiment is to measure this angle.

A scheme of the experiment is shown in fig.4. An incident polarized neutron beam is monochromatized by a Bragg reflection from a perfect Si crystal. It is reflected from a magnetic mirror, and reflected neutrons are analyzed with the similar perfect Si crystal. If the incident neutrons are polarized along magnetization of the mirror, their reflection is total and specular, The reflected neutrons after Bragg diffraction from Si are counted by the detector. The Bragg diffraction is the same as at monochromatization.

If the incident neutrons are polarized against magnetization of the mirror, their reflection is

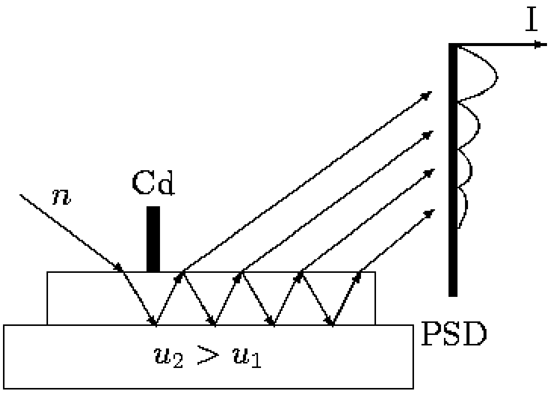

Figure 5: Multiple reflection from two interfaces of sufficiently thick film with potential u_1 on top of a substrate with potential u_2. In geometric optics or ray approximation, when shifted wave fields on the mirror surface do not overlap, reflection can be represented as multiple incoherent reflections from two interfaces. In that case a diffraction pattern can be observed at the position sensitive detector PSD. Every maximum will correspond to beam, which escaped the film after consecutive reflections from the substrate.

not total and direction of reflection deviates from the specular one by some arc seconds. The Si crystal does not diffract them to detector any more, and to increase count rate of the detector, the Si crystal should be rotated by the same angle γ. The closer is $k_\perp$ to critical value $\sqrt{u}$ the larger is γ.

Deviation of reflection direction from the specular one takes place because [16] plain wave constituents $\exp(i\boldsymbol{pr})$ with higher $p_\perp$ have smaller reflection amplitude Eq. (71).

6.2.3 Geometrical and wave optics

In wave optics, when we consider reflection of a plain wave $\exp(i\boldsymbol{pr})$ from a plate of thickness d with interaction potential u, we take into account multiple reflections between the two interfaces [2]. So reflection amplitude $R_d(p_\perp)$ is given by the equation

$$R_d(p_\perp) = R(p_\perp)\frac{1-\exp(2ip'_\perp d)}{1-R^2(p_\perp)\exp(2ip'_\perp d)}, \tag{73}$$

where $p'_\perp = \sqrt{p_\perp^2 - u}$ and $R(p_\perp)$ is given by Eq. (71). The reflectivity in this case is

$$|R_d(p_\perp)|^2 = \left|R(p_\perp)\frac{1-\exp(2ip'_\perp d)}{1-R^2(p_\perp)\exp(2ip'_\perp d)}\right|^2. \tag{74}$$

However, if we deal with a wave packet, we can consider multiple reflections in terms of rays. Then the reflectivity can be calculated as a sum of separate reflectivities

$$|R_d(p_\perp)|^2 = 2|R(p_\perp)|^2\sum_{n=0}^{\infty}\left|\exp(2inp'_\perp d)R^{2n}\right|^2, \tag{75}$$

and the reflection process can be illustrated by fig.5. In this case reflection from two interfaces add incoherently. Of course, the real picture can be intermediate, because in the case of wide wave packet some waves interfere and some not. Investigation of neutron distribution at the position sensitive detector will show how large is interference or how large is a coherence length of the wave packet.

In fact properties of wave packet can be searched with additional parameters in fitting of reflectivities of any multilayer system. The structure of wave packet can be estimated from careful analysis of diffractions pictures of single crystals, because diffraction of some constituent plain waves in the wave packet leads to diffraction of the whole particle with probability corresponding to the weight of the related waves in the wave packet.

7 Resolution of contradiction $A \gg \lambda^2$

To resolve the contradiction that A must be large, and at the same time cannot be large, let's look at the wave function Eq. (7). Though the scattered wave function has the form

$$\psi_s = \int f(\Omega)d\Omega\exp(i\boldsymbol{k}_\Omega\boldsymbol{r}) \tag{76}$$

it does not mean that the scattering probability is

$$dW(\Omega) = |f(\Omega)|^2 d\Omega. \tag{77}$$

It is more correct to represent the integral in the form of integral sum

$$\psi_s = \sum_j f(\Omega_j)\delta\Omega\exp(i\boldsymbol{k}_{\Omega_j}\boldsymbol{r}), \tag{78}$$

where $\delta\Omega$ is some angular quantum.

Then probability to measure by a detector some particles inside a solid angle $\Delta\Omega$, containing $n = \Delta\Omega/\delta\Omega$ angular quanta is

$$dW = \sum_{j=k}^{k+n} |f(\Omega_j)|^2 \delta^2\Omega = \Delta\Omega |f(\Omega_k)|^2 \delta\Omega, \tag{79}$$

where we supposed that $|f(\Omega_j)|^2$ almost does not change in the interval $k \le j \le k+n$.

Now the cross section is

$$d\sigma = \Delta\Omega A |f(\Omega_k)|^2 \delta\Omega, \tag{80}$$

and we can have Large A if $\delta\Omega$ is small. So, if with expression Eq. (7) we want to have

$$d\sigma = \Delta\Omega |b|^2, \tag{81}$$

we must require that $A\delta\Omega = \lambda^2$, so, if $A \approx 10^8\lambda^2$, then $\delta\Omega \approx 10^{-8}$ steradian, and our contradiction is resolved.

8 Conclusion

We have shown that spherical waves are not appropriate for description of scattering. The standard scattering theory is not a theory but a set of logically inconsistent recipes to cook cross sections. The fundamental scattering theory contains an error and therefore cannot be accepted as a proof of validity of SST.

When we try to mend inconsistency of spherical waves, we exclude from them exponentially decaying part and as a result obtain asymptotics containing only scattered plain waves with coefficients, which are probability amplitudes of scattering.

To get a cross section with the help of probability we introduced wave packet with large cross area A, and suggested that there are no scattering if the atom is not embraced by the packet. This suggestion was shown to be incorrect in linear theories, but since we have nothing instead, we accepted this suggestion as an implicit introduction of nonlinearity.

With introduction of A we found a contradiction, that A must be large, and at the same time cannot be large. This contradiction is resolved by introduction of quantization of the scattering angle.

To check all this construction we need a lot of experiments aimed at investigation of wave packet properties of particles, and we appeal to scientific community to start them.

Acknowledgement

Author is grateful to organizing committee of QUANTUM 2010 conference for invitation and support and also to administration of FLNP JINR for giving an opportunity to attend the conference.

9 Illustration of censorship

This paper was submitted to the workshop QUANTUM 2010 (24-29 May 2010) in INRiM (Turin) Italy before the workshop. Proceedings of it was planned in a special issue of the "International Journal of Quantum Information" (IJQI).

Previously I received a report of a referee on 11.06, and corrected the paper in attempt to satisfy some requirements of the referee. The paper presented here is the corrected one.

9.1 Referee 1 report and my reply to it

I present here directly my reply sent on 17.06, in which I cite the referee report point by point. Some words in it I emphasized in bold face. This was my reply.

The referee writes

> 1) The manuscript consists in a critical discussion about the foundations of scattering theory. The author considers the "three scattering theories in non relativistic quantum theory" and claims to have individuated inconsistencies in each of the them and conclude that no proper theory of scattering, despite the success of existing ones in describing experimental data, is available to date.

Thank you it is correct. As for "success of existing ones in describing experimental data" it is really wonderful. I myself continue to use them. However. In my opinion, description of experimental data by existing theories is in some respect phenomenological, as is shown in this paper. So it will be very interesting if we can find a theory without contradictions. It is not interesting to be only pragmatic: "it works, therefore it is true".

> 2) A discussion about foundations of scattering theory is in principle of interest. However, the present manuscript is not well written and **contains several unjustified claims.**

I read out the paper once again and found that it was really not well written. I tried to improve it. I hope I will be able to improve it even more after your next report, and will be grateful for all your suggestions. However, you did not point out what claims are "unjustified", so I do not know how to reply. I will appreciate, if you will do it in your next report.

> 3) It appears that the author have just examined a **limited amount of literature** finding the approach unsatisfactory.

You are right. I am unable to examine "unlimited amount." Does it mean that the paper should be never published?:) If you mean that I missed some important references with different approach, write me them, please, and I will include.

> 4) the literature about neutron scattering is huge and the reference list of this manuscript does not appears to be an exhaustive sampling of the different approaches.

I am sorry, but I do not understand what do you mean.

> 5)The author should include a proposal for an experiment where the claimed inconsistencies become apparent, with a clear indication of what should be measured and in which conditions.

I appealed to scientific community to start research on wave packets. I made references where the corresponding experiments were described. I decided to meet your desire and included some of them here.

> 6) - I could not recognize any reminiscence of Lippmann-Schwinger theorem in the manuscript. How this applies to the author' analysis?

Well, I included one of the equations (see Eq. (47)) known as Lippmann-Schwinger equations. I can derive them all, but tell me, please, what is the most important their feature I must stress upon? Tell me, please, what important feature I missed?

> 7)- What happens to the author' arguments when relativist theory are taken into account?

I deleted the word nonrelativistic from the title. The relativistic theory is S-matrix theory, which follows from Fundamental Scattering Theory, where I also pointed out the contradictions.

> 8) The author makes quite general claims, but then mostly refers to neutron scattering. Perhaps, a more appropriate title, after the revisions outlined above, would be "A critical view to interpretation of neutron scattering data" or similar.

I changed the paper a little bit for to avoid impression that the matter is only related to the neutron scattering. Neutron scattering is used by me only as an example for the sake of concreteness and because I feel myself to be some kind of a specialist in this area. If you not insist very much I would prefer to save this already changed title.

Finally I would like to assure you, that my goal is not to overthrow quantum mechanics. My findings about contradictions are absolutely accident. I tried to see whether it is possible to avoid introduction of some artificial scale L. We all learn everything from textbooks, and if have some doubts we attribute them to our poor knowledge. Later we became so much accustomed to what we learnt that we consider our knowledge as a perfect one. It is a so common psychological effect. You are not the first referee, who resolutely rejects the paper immediately. I beg you, please, read out your report once again, and point out the so called "unjustified claims".

On 20.07 I received the complete rejection of my paper. The first referee only added few lines

> I carefully read the author's reply and the revised version of this manuscript. I appreciated the author's revision but I still believe that his **polemic attitude make the manuscript of little use** for the community in order to start a debate, which I sincerely believe is the main aim of the author. I'm anyway ready to admit that I missed something of this manuscript and thus I believe that a second opinion is in order.

It is seen that the referee do not advise publication.

9.2 The second referee report

The second referee report was long and boring even for me myself. So I will cite only small parts of it illustrating his attitude to this paper and comment them in italic.

> The author ... criticizes the lack of wave packets, the quantization of scattering angle and non-linearity effects. The first point seems justified, the second and third one not because there is a continuous eigen-value solution for the scattering angle and non-linear effects have not been observed yet.

There are no discussion of my arguments as if they were absent

> The author starts with the scattering from a rigidly bound nucleus and criticizes that the scattered wave is described by a spherical wave which exists only at distance from the nucleus. This is not a drawback since ...

And after that he teaches me what is said in textbooks, which I miss here

> The standard scattering theory is criticized with **the same arguments**. Here also the mistake happened that a plane wave is assumed as scattered wave.

It looks that he is unable to understand arguments. He cannot differentiate them.

> In Chap. 5 he author jumps into non-linear Schrodinger equation by assuming a non-spreading wave packet. This is interesting but not physical since **spreading is observed** in many situations when t=0 is defined and therefore the time-dependent Schrodinger equation has to be used.

It is interesting where did he observe spreading: in an experiment or on a paper?

> Then he assumes that the wave packet is an intrinsic property of a particle itself which is in strong contradiction to quantum optics laws

He is absolutely illiterate man. He cannot read. He found the words "non spreading" and immediately remembered the textbooks theorem.

> It should also made clear that a wave packet **does not spread in free propagation** since **time factorizes** and the stationary Schrodinger equation does not give spreading since the solution is an eigen-value solution

He absolutely does not understand what he writes. If we have eigen-value solution, then we have a bound state. Where it is then the free propagation?

We want nevertheless to cite some sample of his writing to show how boring it is to read him.

> The starting comment to Chap. 6 "We cannot deduce A, but we can explore its properties" is **completely misleading** and should be removed since A can be determined rather precisely (see above) and therefore the discussion whether s is small or large is irrelevant because it is as defined by the beam preparation. In the discussion of related experiments he mentions several effects which are proportional to the packet spread in momentum space (thin plate transmission, Goos-Hanchen effect etc). These effects exist but can be explained directly by the non-spreading wave-packets from the stationary Schrodinger equation because more wave components $k > k_o$ have higher transmission. The statement s =prop.k is purely technically justified because it becomes more difficult to monochromatize and collimate higher energy neutrons (particles).

The following sentence shows that he is absolutely incompetent:

> ultra-cold neutrons can have packet dimensions smaller than the wave length.

And his final conclusion is

> Although I welcome **competent** discussions about quantum and scattering theories I believe the present manuscript does not contribute to make the issue clearer mainly because all effect described as crucial experiments are well described by the standard theory as well. A major revision would be necessary to make the manuscript publishable.

After such a report it is clear that the paper will be never accepted because everything which goes outside the textbooks is characterized as incompetent.

Once on my lecture about these contradictions a man, sitting on the first raw was commented "Oh everything is not like that!" "The spherical waves are perfect!" "You should better learn the textbooks."

I made a pause and told to him. "Sir, if you were my student, I would put you the best mark at your exam. But, you see, when you became a researcher you acquire a professional disease. This disease has the name "DOUBT". I should say that I envy you. You are completely healthy man."

"I don't understand, whether you praised or abused me." grumbled he.

I am sure that the referee also will not understand that.

I did not appeal against such a report. It is really impossible, and the editor will tell that it is too late. I had already had such an experience with the other paper on EPR paradox: see http://arxiv.org/abs/1007.3202. I am also sure that to my complains about referee the editor will reply

> The present referee is a very serious and expert physicist.

9.3 To readers

Dear readers, if you will be interested to read my paper, if you will have some questions, if you want to read the full referee report, write me, please ignatovi@nf.jinr.ru Your letters are the most important to me. The physics is very interesting notwithstanding that it became like ideological autocracy now.

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