ON EPR PARADOX, BELL'S INEQUALITIES AND EXPERIMENTS THAT PROVE NOTHING

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EPR paper [1] contains an error. Its correction leads to a conclusion that position and momentum of a particle can be defined precisely simultaneously, EPR paradox does not exist and uncertainty relations have nothing to do with quantum mechanics. Logic of the EPR paper shows that entangled states of separated particles do not exist and therefore there are no nonlocality in quantum mechanics. Bell's inequalities are never violated, and results of experiments, proving their violation, are shown to be false. Experiments to prove absence of nonlocality are proposed where Bell's inequalities are replaced by precise prediction. Interpretation of quantum mechanics in terms of classical field theory is suggested.

Keywords: EPR paradox; entangled states; Bell's inequalities

1. Introduction

In EPR paper [1] it is shown that on one side from the common sense logic it follows that a particle can have a position and momentum simultaneously, but on the other side quantum mechanics forbids it because position and momentum operators do not commute. This contradiction is the essence of the EPR paradox. We will show that the contradiction arises from incorrect definition of the value of a physical quantity as an eigen value of the corresponding operator. Such a definition leads to an elementary error. Correction of this error leads to redefinition of a physical quantity and to resolution of the paradox. But let's start from beginning.

2. Error in the EPR paper

In [1] it is said

> If ψ is an eigenfunction of the corresponding operator A, that is, if
>
> $$\psi' \equiv A\psi = a\psi, \qquad ([1]1)$$
>
> where a is the number, then the physical quantity A has with certainty the value a whenever the particle is in the state given by ψ.
>
> In particular, the momentum p is defined for the wavefunction represented by a plane wave
>
> $$\psi = \exp(2\pi i p_0 x/h), \qquad ([1]2)$$
>
> since the eigenvalue of the momentum operator $\hat{\boldsymbol{p}} = (h/2\pi i)d/dx$ for this wavefunction is p_0. Thus, in the state given by Eq. ([1]2), the momentum has certainly the value p_0. It thus has meaning to say that the momentum of the particle in the state given by Eq. ([1]2) is real.

In such a state, however, we have no information about the particle's position. According to EPR [1] we can

> only say that the relative probability that a measurement of the coordinate will give a result lying between a and b is
>
> $$P(a,b) = \int\limits_a^b |\psi(x)|^2 dx = b - a. \qquad ([1]6)$$

Stop! Here we see the error. The value $P(a,b)$ is not a probability, because it is not dimensionless and because it is not normalizable.

All the specialists and textbooks on quantum mechanics ignore this error. They use a modified plain wave $\exp(ikx)/\sqrt{L}$ instead of $\exp(ikx)$ with some (they add "large") linear scale L. Then the EPR probability will look $(b-a)/L$, so after such innocent correction it becomes dimensionless. The problem of normalization is solved by requirement that all the space is limited by the scale L, so $|b-a| \leq L$.

Such a trick can satisfy only students, who do not understand it but have nothing to do as to believe their teachers. However, let us ask ourselves: what does this mean? Some of guru in quantum mechanics tell that it is possible to impose periodic boundary conditions at the ends of the interval L. However periodicity transforms the plain wave into a Bloch wave function $\psi(x) = \exp(iqx)\varphi(x)$, where $\varphi(x)$ is a periodic function with period L. The Bloch wave function is not normalizable and it is not an eigen function of the momentum operator. Therefore the system in the periodic space has no momentum and no position.

Some other guru in quantum mechanics tell that we can imagine ourselves in a limited but large space. However, the requirement $|b-a| \leq L$ means that this space is limited by impenetrable walls, and impenetrability conditions transform the plain wave $\exp(ikx)$ into a combination of real functions $\cos(kx)$ and $\sin(kx)$ neither of which is an eigen function of the momentum operator. Therefore the system in the limited space has no momentum.

3. Correction of the error

We can correct the EPR error only if we replace plain wave by a wave packet. However no wave packet is an eigen function of the momentum operator, therefore no system has a value for such a physical quantity as momentum. If we do not accept such a conclusion, we must redefine the notion of the value of a physical quantity. We can define position and momentum of particles as expectation values:

$$x = \int \psi^+(x')\hat{x}\psi(x')dx', \qquad p = \int \psi^+(x')\hat{p}\psi(x')dx', \tag{1}$$

then they exist simultaneously and EPR paradox is resolved. Noncommutativity of operators $\hat{x}$ and $\hat{p}$ does not preclude simultaneous precise definitions of x and p according to Eq. (1), therefore uncertainty relations have nothing to do with quantum mechanics. They are valid in quantum mechanics because they are valid in any branch of physics dealing with functions. Uncertainty relations is a mathematical theorem, which relates range of any function to the range of its Fourier image.

The guru in quantum mechanics immediately react to relations Eq. (1). They tell that with such a definition we have dispersions

$$\Delta x^2 = \int \psi^+(x')(\hat{x}-x)^2\psi(x')dx', \qquad \Delta p^2 = \int \psi^+(x')(\hat{p}-p)^2\psi(x')dx'. \tag{2}$$

Therefore position and momentum are defined not precisely, but with some uncertainty.

Our reply to this objection is: this uncertainty is not statistical one, but it is a natural uncertainty of definition. For illustration, lets look at an any object of nonzero size. Can we say what is its position? Yes, we can say, but position point is a matter of definition. It can be the center of gravity, or geometrical center, or the closest point to an observer. For every extended in space object we can also find a dispersion of the previously defined position, and this dispersion will characterize the form and the size of the object.

The same is in quantum mechanics. Every wave packet is an extended object. We can define its position, for example, with Eq. (1), and its dispersion Eq. (2) characterizes its property — the width of the packet. The same is related to the momentum of the particle.

At this point the guru claim, that the whole wave function, whether it is a plain wave or a wave packet, determine only statistical properties of an object, therefore, the width of the wave packet is also a statistical dispersion. We reject this claim, and show at the end of this article that the wave function can be interpreted as a classical field, therefore it is a material object, and its size can be well measured.

4. Entangled states of separated particles do not exist

The EPR paper considers two particles which interacted at some past moment and then flew far apart. Notwithstanding of how large is the distance between them they have a common entangled wave function

$$\Psi(x_1, x_2) = \sum_n \phi_n(x_1)u_n(x_2). \tag{3}$$

According to EPR logic, if particle 1, after measurement is found in the state $\phi_m(x_1)$, then the state of the particle 2 is $u_m(x_2)$. But the particle 2 is far away from 1 and is not perturbed by measurements of 1, therefore the particle 2 had the state $u_m(x_2)$ before the measurement.

Following this logic we immediately conclude that the wave function of two particles before the measurement was not Eq. (3), but it was a simple product

$$\Psi(x_1, x_2) = \phi_m(x_1)u_m(x_2), \tag{4}$$

and the measurement only revealed what product it really was. So the entangled state Eq. (3) represents only a list of possible states for separated particles. The total sum Eq. (3) is forbidden in quantum mechanics like forbidden are the exponentially growing solutions of the Schrödinger equation.

5. Bohm-Aharonov version of the EPR entangled state

Bohm-Aharonov [2] considered EPR paradox and entanglement in terms of spin operators of 1/2 spin particles, and photon's polarizations. We will show that there are neither paradox, nor entanglement in both cases.

5.1. *Entangled spin state*

In [2] it is considered the decay of a singlet state $|\psi(1,2)\rangle$ of a molecule consisting of two spin 1/2 atoms. In the molecule before the decay the spinor state of the two atoms, denoted by digits 1,2, is

$$|\psi_0(1,2)\rangle = \frac{1}{\sqrt{2}}\Big(|\psi_{-\boldsymbol{a}}(2)\psi_{\boldsymbol{a}}(1)\rangle - |\psi_{\boldsymbol{a}}(2)\psi_{-\boldsymbol{a}}(1)\rangle\Big), \tag{5}$$

where $|\psi_{\pm\boldsymbol{a}}(i)\rangle$ is a spinor of particle i polarized along or opposite to an arbitrary direction denoted by a unit vector $\boldsymbol{a}$. According to [2] this state remains unchanged after decay, so spin states of separated particles become entangled. It means that if after separation of two particles, when

> they cease to interact, any desired component of the spin of the first particle (A) is measured. Then, because the total spin is still zero, it can be concluded that the same component of the spin of the other particle (B) is opposite to that of A.
>
> If this were a classical system there would be no difficulties, in interpreting the above result, because all components of the spin of each particle are well defined at each instant of time.
>
> In quantum theory, a difficulty arises, in the interpretation of the above experiment, because only one component of the spin of each particle can have a definite value at a given time. Thus, if the x component is definite, then y and z components are indeterminate and we may regard them more or less as in a kind of random fluctuation.

Here we must to make some digression on measurement and measurables.

5.2. *No measurable was ever measured*

It is common in quantum mechanics to tell that every hermitian operator is a measurable. We claim that no of them was ever measured. Experimental devices consist of a filter (analyzer) and a detector which counts the transmitted particle. To explain how it works for spin measurements we will make a small excursus into spinorial mathematics [3].

Every spinor particle is described by a mathematical construction — spinor, and every spinor corresponds to a precisely defined classical object — spin arrow, which is a vector with some absolutely precisely defined direction. For a normalized arbitrary spinor

$$|\psi\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix}, \tag{6}$$

with $|\alpha|^2 + |\beta|^2 = 1$ the spin arrow $\boldsymbol{s}$ is the unit vector with components

$$\boldsymbol{s} = (s_x, s_y, s_z) = \langle\psi|\boldsymbol{\sigma}|\psi\rangle, \tag{7}$$

where $\boldsymbol{\sigma}$ is the vector of Pauli matrices $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$.

A particle polarized along a unit vector $\boldsymbol{a}$ is described by the normalized spinor, which can be represented as [3]

$$|\psi_{\boldsymbol{a}}\rangle = \frac{I + \boldsymbol{\sigma}\cdot\boldsymbol{a}}{\sqrt{2(1+\boldsymbol{a}\cdot\boldsymbol{b})}}|\psi_{\boldsymbol{b}}\rangle, \tag{8}$$

where $|\psi_{\boldsymbol{b}}\rangle$ some other normalized spinor with spin-arrow along the unit vector $\boldsymbol{b}$. A change of vector $\boldsymbol{b}$ changes the phase factor of $|\psi_{\boldsymbol{a}}\rangle$, but does not change direction of its spin arrow.

If the filter is directed along a unit vector $\boldsymbol{A}$, it does not measure component of the spin arrow $\boldsymbol{a}$ along $\boldsymbol{A}$ of the spinor $|\psi_{\boldsymbol{a}}\rangle$. The filter only transmits the particle with the probability

$$w(\boldsymbol{a},\boldsymbol{A}) \propto \langle\psi_{\boldsymbol{a}}|\frac{I+\boldsymbol{\sigma}\cdot\boldsymbol{A}}{2}|\psi_{\boldsymbol{a}}\rangle, \tag{9}$$

or reflects it with probability $\approx 1-w(\boldsymbol{a},\boldsymbol{A})$. (Imagine, for instance, a magnetic mirror, which transmits only neutrons polarized oppositely to its magnetization.) Below we will consider only ideal filters for which relation Eq. (9) can be replaced by equality:

$$w(\boldsymbol{a},\boldsymbol{A}) = \langle\psi_{\boldsymbol{a}}|\frac{I+\boldsymbol{\sigma}\cdot\boldsymbol{A}}{2}|\psi_{\boldsymbol{a}}\rangle = \frac{1+\boldsymbol{a}\cdot\boldsymbol{A}}{2}. \tag{10}$$

However in a single measurement you can tell nothing about probabilities. You can only tell, that the particle is transmitted or not, and if it is transmitted, it after transmission is in the state $|\psi_{\boldsymbol{A}}\rangle$. This change of state from initially not known polarization to the state with spin arrow along $\boldsymbol{A}$ happens not because of Copenhagen's reduction of the wave packet, but because of a dynamical action of the filter.

Of course, if you have a beam of particles, ideally polarized along a fixed vector $\boldsymbol{a}$, then a statistical experiment with many particles reveals probability Eq. (10), from which you can find $\boldsymbol{a}\cdot\boldsymbol{A}$. But if you prepared such a beam, your measurement with analyzer $\boldsymbol{A}$ is not needed. Such a measurement only helps to control how well the beam was prepared.

5.3. *Correlations of spinor particles*

Now we can go back to Eq. (5) and consider what would happen if we were able to apply a simultaneous analyzer to two particles in the molecule before the decay and after it [4]. Before the decay the hypothetical measurement will give the probability of coincident count of two particles equal to

$$R_0(\boldsymbol{A},\boldsymbol{B}) = \langle\psi(1,2)|\left(\frac{1+\boldsymbol{\sigma}\cdot\boldsymbol{A}}{2}\right)_1\left(\frac{1+\boldsymbol{\sigma}\cdot\boldsymbol{B}}{2}\right)_2|\psi(1,2)\rangle = \frac{1-(\boldsymbol{A}\cdot\boldsymbol{B})}{4}, \tag{11}$$

and it is seen that $R_0(\boldsymbol{A},\boldsymbol{A})=0$, and $R_0(\boldsymbol{A},-\boldsymbol{A})=0.5$. To calculate this expression we substituted Eq. (5), (8) and chose $\boldsymbol{b}\perp\boldsymbol{a}=0$.

With Eq. (11) we can calculate [5] the correlation $E(\boldsymbol{A},\boldsymbol{B})$ used by Bell [6]

$$E_0(\boldsymbol{A},\boldsymbol{B}) = R_0(\boldsymbol{A},\boldsymbol{B}) - R_0(-\boldsymbol{A},\boldsymbol{B}) - R_0(\boldsymbol{A},-\boldsymbol{B}) + R_0(-\boldsymbol{A},-\boldsymbol{B}) = -(\boldsymbol{A}\cdot\boldsymbol{B}), \tag{12}$$

and show that inequalities

$$|R_0(\boldsymbol{A},\boldsymbol{B}) - R_0(\boldsymbol{A},\boldsymbol{C})| \le 1 - R_0(\boldsymbol{A}',\boldsymbol{B}) - R_0(\boldsymbol{A}',\boldsymbol{C}), \tag{13}$$

$$|E_0(\boldsymbol{A},\boldsymbol{B}) - E_0(\boldsymbol{A},\boldsymbol{C})| \le 1 + E_0(\boldsymbol{B},\boldsymbol{C}), \tag{14}$$

can be violated for atoms inside the molecule. If the entangled state Eq. (5) does not change after decay, then these inequalities can be violated in measurement with separated particles. Many experiments were performed to check these inequalities. They "prove" that inequalities are really violated, therefore entangled states of separated particles exist, and quantum mechanics is nonlocal theory. We will show that this prove is false.

We proceed from the assumption that entangled states of separated particles are forbidden. Then decay of the molecule is a transition from the state Eq. (5) to the state of outgoing particles

$$|\psi(1,2)\rangle = |\psi_{\boldsymbol{n}}(1)\rangle|\psi_{-\boldsymbol{n}}(2)\rangle, \tag{15}$$

i.e. two particles are going away in a product state with opposite directions of polarizations defined by an arbitrary unit vector $\boldsymbol{n}$. This is the classical hidden parameter λ, and we can introduce probability

$$\rho(\lambda)d\lambda = \frac{d\Omega_{\boldsymbol{n}}}{4\pi}, \tag{16}$$

which determines isotropic distribution of directions $\boldsymbol{n}$ over solid angle Ω.

Probability of coincident counts for the first particle transmission through analyzers $\boldsymbol{A}$ and of the second particle transmission through analyzers $\boldsymbol{B}$ is

$$R_c(\boldsymbol{n},\boldsymbol{A},\boldsymbol{B}) = \langle\psi_{\boldsymbol{n}}(1)|\frac{I+\boldsymbol{\sigma}\cdot\boldsymbol{A}}{2}|\psi_{\boldsymbol{n}}(1)\rangle\langle\psi_{-\boldsymbol{n}}(2)|\frac{I+\boldsymbol{\sigma}\cdot\boldsymbol{B}}{2}|\psi_{-\boldsymbol{n}}(2)\rangle = \frac{(1+\boldsymbol{n}\cdot\boldsymbol{A})(1-\boldsymbol{n}\cdot\boldsymbol{B})}{4}. \tag{17}$$

After averaging of it over Eq. (16) we get

$$R_c(\boldsymbol{A},\boldsymbol{B}) = \int \frac{d\Omega}{4\pi} R_c(\boldsymbol{n},\boldsymbol{A},\boldsymbol{B}) = \int \frac{d\Omega}{4\pi} \frac{(1+\boldsymbol{n}\cdot\boldsymbol{A})(1-\boldsymbol{n}\cdot\boldsymbol{B})}{4} = \frac{1-(\boldsymbol{A}\cdot\boldsymbol{B})/3}{4}, \tag{18}$$

and respectively

$$E_c(\boldsymbol{a},\boldsymbol{b}) = -\frac{(\boldsymbol{A}\cdot\boldsymbol{B})}{3}. \tag{19}$$

These expressions do not violate inequalities Eq. (13) and (14) respectively.

5.4. *Experiment and inequalities*

Let's compare Eq. (18) and Eq. (11). The first one can be represented as

$$R_c(\boldsymbol{A},\boldsymbol{B}) = \frac{1-(\boldsymbol{A}\cdot\boldsymbol{B})/3}{4} = \frac{1}{6} + \frac{1}{3}\left(\frac{1-\boldsymbol{A}\cdot\boldsymbol{B}}{4}\right) = \frac{1}{6} + \frac{1}{3}R_0(\boldsymbol{A},\boldsymbol{B}), \tag{20}$$

so $R_c(\boldsymbol{A},\boldsymbol{A})/R_c(\boldsymbol{A},-\boldsymbol{A}) = 0.5$. If you exclude the constant term, as a background, and by normalization exclude factor $1/3$, then you transform Eq. (20) into Eq. (11), and this way "prove" that inequalities Eq. (13) are violated. We do not make a reference to experiments with spin 1/2 particles, because they are not sufficiently precise and therefore are not considered as a direct prove of nonlocality in quantum mechanics. The most popular are experiments with photons, and now we will analyze correlation of photon pairs.

5.5. *Entangled photon pair*

Let's consider experiment [7] with cascade decay of an excited calcium atom. The radiated photons, are supposed to compose an entangled pair whose polarization state can be represented by the function

$$|\psi_0(1,2)\rangle = \frac{1}{\sqrt{2}}[|x,-x\rangle + |y,-y\rangle], \tag{21}$$

where, for example, the state $|x,-x\rangle$ corresponds to polarization of the photons 1,2 along $\pm x$ axis.

Transmission of a photon in a state $|\boldsymbol{a}\rangle$ through analyzer directed along $\boldsymbol{A}$ goes with probability

$$w(\boldsymbol{a},\boldsymbol{A}) = |\langle\boldsymbol{A}||\boldsymbol{a}\rangle|^2 = (\boldsymbol{A}\cdot\boldsymbol{a})^2. \tag{22}$$

Therefore coincidence count rate for two photons in the state Eq. (21) when two analyzers are directed along vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ is

$$R_0(\boldsymbol{A}.\boldsymbol{B}) = |\langle\boldsymbol{A},\boldsymbol{B}||\psi_0(1,2)\rangle|^2 = \frac{1}{2}|A_xB_x + A_yB_y|^2 = \frac{1}{2}(\boldsymbol{A}\cdot\boldsymbol{B})^2 = \frac{1+\cos(2\phi)}{4}. \tag{23}$$

where ϕ is the angle between two unit vectors $\boldsymbol{A}$ and $\boldsymbol{B}$. This function can violate inequality Eq. (13).

Again we proceed from the assumption that entangled states of separated particles are forbidden. Then two outgoing photons have the state

$$|\psi_c(1,2)\rangle = |\boldsymbol{n}\rangle|-n\rangle, \tag{24}$$

i.e. two particles are going away in a product state with opposite directions of polarizations defined by a unit vector $\boldsymbol{n}$, which is perpendicular to direction of their flight. We will consider a simplified version of the experiment, suggesting that two photons fly along $\pm z$ directions. Therefore the vector $\boldsymbol{n}$ of polarization lies in the (x,y) plane. This vector is the classical hidden parameter λ, and we can introduce probability

$$\rho(\lambda)d\lambda = \frac{d\varphi_{\boldsymbol{n}}}{2\pi}, \tag{25}$$

which determines isotropic distribution of directions $\boldsymbol{n}$ in the (x,y) plane.

Probability of coincident counts for the first photon transmitted through analyzers $\boldsymbol{A}$ and of the second photon transmitted through analyzers $\boldsymbol{B}$, when both vectors $\boldsymbol{A}$, $\boldsymbol{B}$ are in (x,y) plane is

$$R_c(\boldsymbol{n},\boldsymbol{A},\boldsymbol{B}) = (\boldsymbol{n}\cdot\boldsymbol{A})^2(\boldsymbol{n}\cdot\boldsymbol{B})^2. \tag{26}$$

After averaging of it over Eq. (25) we get

$$R_c(\boldsymbol{A},\boldsymbol{B}) = \int \frac{d\varphi}{2\pi} R_c(\boldsymbol{n},\boldsymbol{A},\boldsymbol{B}) = \frac{1}{4}\left(1 + \frac{\cos(2\phi)}{2}\right), \tag{27}$$

where ϕ is the angle between vectors $\boldsymbol{A}$ and $\boldsymbol{B}$. This expression does not violate inequality Eq. (13). It can be also represented as

$$R_c(\boldsymbol{A},\boldsymbol{B}) = \frac{1}{8} + \frac{1}{2}\left(\frac{1+\cos(2\phi)}{4}\right) = \frac{1}{8} + \frac{1}{2}R_0(\boldsymbol{A},\boldsymbol{B}), \qquad (28)$$

so $R_c(\boldsymbol{A},\perp \boldsymbol{A})/R_c(\boldsymbol{A},\boldsymbol{A}) = 1/3$ and not zero, as it follows from Eq. (23). If you exclude the constant term, as a background, and by normalization exclude factor 1/2, then you transform Eq. (27) into Eq. (23), and this way "prove" that inequality Eq. (13) is violated. **It is this way the experimentalists** [7,8,9,10] **"prove" nonlocality of the quantum mechanics**, which is enthusiastically praised, for example, in [11]:

> The concept of entanglement has been thought to be the heart of quantum mechanics. The seminal experiment by Aspect et al. [7] has proved the "spooky" nonlocal action of quantum mechanics by observing violation of Bell inequality with entangled photon pairs.

Now we see, that to prove nonlocality it is necessary to check the prediction Eq. (27), and for that it is necessary to learn how to separate really accident coincidences from those that follow from the constant term in Eq. (27).

6. Classical interpretation of the quantum mechanics

We think that the wave function of particles is their real field, with which they interact with environment. The most spectacular is the field of the de Broglie's singular wave packet. The motion of a particle of mass m can be described by the system of classical equations

$$m\frac{d^2\boldsymbol{r}(t)}{dt^2} = \boldsymbol{F}(\psi(\boldsymbol{r}(t),t)), \qquad (29)$$

$$(\partial_t^2 - \Delta)\psi(\boldsymbol{r},t) = j(\boldsymbol{r}(t),t), \qquad (30)$$

where the first, Newton, equation contains a force which depends on field $\psi(\boldsymbol{r},t)$ at the position of the particle, and the second equation is the equation for a field which is created by particle and depends on boundary conditions on the surrounding bodies. The classical system Eq. (29), (30) has nothing to do with Bohmian mechanics, which is not a mechanics, but a method how to solve Schrödinger equation. We think that quantum mechanics is an ingenious theory which replaced the complicated nonlinear classical system with a linear Schrödinger equation. However the price for simplicity is indeterminism and probabilities.

The system Eq. (29), (30), when solved will show how interference appears in classical mechanics [4].

One of the most important problems in quantum mechanics is reflection of a particles from a semitransparent mirror. It can be considered as a bifurcation within some scheme of classical stochastic dynamics.

7. Conclusion

We have shown that EPR paradox does not exist, that uncertainty relations have nothing to do with quantum mechanics, that entangled states do not exist, and experiments with cascade decay of excited calcium atom do not prove nonlocality of quantum mechanics. We did not analyze here experiments with parametric down conversion of photons because of volume restriction. Some of them were already analyzed in [4], and it was shown that they also do not prove nonlocality of quantum mechanics. Since entangled states of separated particles are forbidden in quantum mechanics, therefore quantum teleportation and quantum cryptography do exist only on paper.

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